

ANSWER ALL QUESTIONS (30 Marks)

1. a) Find the value of c satisfying the conclusion of Rolle's Theorem for

$$f(x) = x^3 - 3x^2 + 2x + 2 \quad (5 \text{ marks})$$

b) Given $f(x) = x^2$ for $0 \leq x \leq 3$. Find the number x between 0 and 3 which satisfies the conclusion of the mean value theorem for definite integrals. (5 marks).

2) a) $\int_0^{\pi} x \sec^2 x \, dx$ (5 marks)

b) $\int \cos^2 x \sin^4 x \, dx$ (5 marks)

c) $\int \frac{\sqrt{x^2-9}}{x} \, dx$ (5 marks)

3) A function $f(x, y)$ is harmonic if $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$. Prove that

$f(x, y) = e^{-x} \cos y + e^{-y} \cos x$ is harmonic. (5 marks)