THE MOMBASA POLYTECHNIC UNIVERSITY COLLEGE

DEPARTMENT OF ELECTRICAL & ELECTRONIC ENGINEERING

Faculty of Engineering and Technology

THIRD YEAR / FIRST SEMESTER EXAMINATIONS FOR THE DEGREE

OF BACHELOR OF SCIENCE IN ELECTRICAL & ELECTRONIC ENGINEERING

EEE 2304 ELECTROMAGNETICS II

AUGUST 2012 SERIES **TIME: 2 HOURS**

INSTRUCTION TO CANDIDATES

Answer question **ONE** and any other **TWO** questions.

1.(a) Write down and explain the practical significance of the following Maxwell's equations

(i) Faraday's

(ii) Ampere's law (4 marks)

(b) The magnetic field in free space is given by

$$H = H_m \cos(wt - \beta z)\ a_y$$

(i) Determine the corresponding electric field from Ampere's law.

(ii) Demonstrate the duality between Faraday's and Ampere's laws by using the result obtained in b(i) to obtain the magnetic field indicated above. (12 marks)

(c) With appropriate sketches show that the energy contained in electric and magnetic fields are given by the following expressions respectively

(i) $\frac{1}{2}\varepsilon_0 \bar{E}^2$

(ii) $\frac{1}{2}\mu_0 \bar{H}^2$ (8 marks)

(d) State the Poynting's theorem and give the physical meaning of the Poynting vector. (3 marks)

(e) A wave travelling in sea water which is lossy has its electric and magnetic fields given in a rectangular coordinate system as

$$\bar{E} = 10e^{-4z}\cos(wt - 4z)\ a_x V/m$$

$$\bar{H} = 7.15e^{-4z}\cos(wt - 4z - \pi/4)\ a_y\ A/m$$

Determine the total power exiting a cube surface having sides of length 2m and corners at (0,0,0), (2m,0,0), (0,0,2m), (2m,0,2m), (0,2m,2m), (2m,2m,2m), (0,2m,0), and

(2m,2m,0). Assume the conductivity of sea water $\delta = 80$ and the frequency of the wave is 1.5 KHz (6 marks)

2.(a) Determine using the phasor method, whether the folowing field vectors satisfy Faraday's and Ampere's laws in free space.

$$\bar{E} = 12.246 \cos\left(\omega t - \left(\frac{8\pi}{3}\right) z\right) a_x$$

$$\bar{H} = 0.0325 \cos\left(\omega t - \left(\frac{8\pi}{3}\right) z\right) a_y$$

(13 marks)

(b) A plane electromagnetic wave having a frequency of 10 MHz has an average Poynting vector of 1 W/m^2. If the medium is lossless with ε_0 = 3 and μ_0 = 2, find:

(i) The velocity of propagation

(ii) Wavelength

(iii) Impedance of the medium

(iv) rms value of the electric field (4 marks)

(c) Assuming that seawater has a conductivity of $20\, S/m$, an $\varepsilon_r = 81$, determine the frequency at which the conduction current is 10 times the displacement current.

(3 marks)

3.(a) (i) Explain the concept of phase shift in the transmission of an electromagnetic wave through a given medium.

(ii) Explain one consequence of phase shift in relation to signal transmission.

(2 marks)

(b) The radiated electric field at a sufficiently large distance from a dipole antenna is of the form

$$\bar{E} = \frac{E_0}{r} sin\theta cos\,(\omega t - \beta r)\, a_\theta$$

Determine

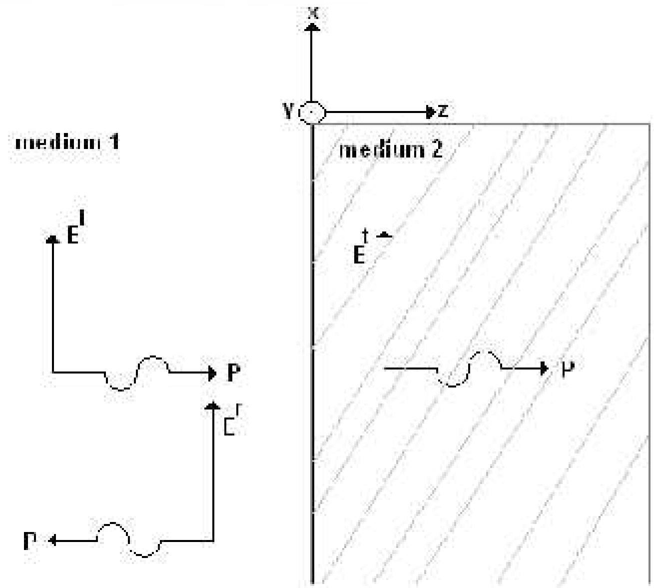
(i) The magnetic field

(ii) The total power radiated by the antenna. (10 Marks)

(c) With reference to fig. Qu.3(c), medium 1 is air, and medium 2 is Teflon ($\mu_{r1} = 1$). Assume the Teflon is infinitely thick so that no reflected waves exist in the Teflon.

Write the time-domain expressions for all the fields in these two media if the transmitted electric field in medium 2 is measured as

$$E^t = 10\cos\left(wt - \left(\frac{8\pi}{3}\right)z\right)a_x$$ (8 marks)



4.(a) Given the Maxwell's equations

$$\nabla X \hat{E} = -j\omega\mu\hat{H}$$

$$\nabla X \hat{H} = \hat{J} + j\omega\varepsilon\hat{E}$$

Prove that $\hat{E}_x = \hat{E}_m{}^{+} e^{-j\beta z} + \hat{E}_m{}^{-} e^{+j\beta z}$

$$\hat{H}_y = \hat{H}^{+} e^{-j\beta z} + \hat{H}^{-} e^{+j\beta z}$$

Explain all your steps and assumptions carefully. (20 marks)

5.(a) (i) Describe essential physical and electrical features of waveguides that make them so useful for propagating RF signals at microwave frequencies.

(ii) State the frequency range of operation of wave guides.

(iii) Explain the relationship between a given energy mode and the waveguide cut-off frequency.

(iv) Distinguish between dominant and degenerate modes in wave guides. (9 marks)

(b) An air-filled rectangular waveguide has cross-sectional dimensions:

$$x = 8cm \qquad y = 4cm$$

(i) Find the cut-off frequency for the following modes, TE_{10}, TE_{20}, TE_{01} TE_{02}, TE_{11} TE_{12}, and TE_{21} modes.

(ii) Determine the dominant mode

(iii) Determine the degenerate modes (11 marks)

Solution set

1.(a) (i) Faraday's law in integral form

$$\oint_c \bar{E}\,.\,\bar{dl} = -\frac{\partial}{\partial t}\int_s \bar{B}\,.\,\bar{ds} \qquad \text{(1 mark)}$$

The line integral of the electric field intensity vector along a contour c is equivalent to the rate of change of magnetic flux cutting through a surface bounded by the contour c. (1 mark)

Ampere's law in integral form:

$$\oint_c \bar{H}\,.\,\bar{dl} = \int_s \bar{J}\,.\,\bar{ds} + \frac{\partial}{\partial t}\int_s \bar{D}\,.\,\bar{ds} \qquad \text{(1 mark)}$$

The line integral of the magnetic field intensity vector around a contour c is equal to the sum of conduction current and the electric density cutting through a surface bounded by the contour c. (1 mark)

(b) (i)

$$\nabla X\bar{H} = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right) a_x + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right) a_y + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right) a_z \qquad \text{(1 mark)}$$

$$= -\frac{\partial H_y}{\partial z} a_x$$

$$= -\frac{\partial(H_m \cos(wt-\beta z)}{\partial z} a_x \qquad \text{(1 mark)}$$

$$= -\beta H_m \sin(wt-\beta z)\, a_x \qquad \text{(1 mark)}$$

Since the curl has only an x component, the right-hand side of Ampere's law can only have an x component, so we obtain

$$-\beta H_m \sin(wt-\beta z)\, a_x = \varepsilon_0 \frac{\partial E_x}{\partial t}\, a_x \qquad \text{(1 mark)}$$

Integrating RHS $E_x = \int -\beta H_m \sin(wt-\beta z)\, dt\;\; a_x = \frac{\beta H_m}{w\varepsilon_0}\cos(wt-\beta z)a_x$ (1 mark)

(ii) $E_x = \frac{\beta H_m}{w\varepsilon_0}\cos(wt-\beta z)a_x \quad H_y = ?$

$$\nabla X\bar{E} = \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) a_x + \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right) a_y + \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right) a_z \qquad \text{(1 mark)}$$

$$\nabla X\bar{E} = \frac{\partial E_x}{\partial z} a_y = \frac{\partial\left(\frac{\beta H_m}{w\varepsilon_0}\cos(wt-\beta z)\right)}{\partial z}\; a_y \qquad \text{(1 mark)}$$

$$\nabla X\bar{E} = \frac{\beta^2 H_m}{w\varepsilon_0}\sin(wt-\beta z)\;\; a_y \qquad \text{(1 mark)}$$

Since the curl has only a y component, the right-hand side of Faraday's law can only have a y component, so we obtain

$$\frac{\beta^2 H_m}{w\varepsilon_0}\sin(wt-\beta z)\ a_y = -\frac{\partial B_y}{\partial t}$$ (1 mark)

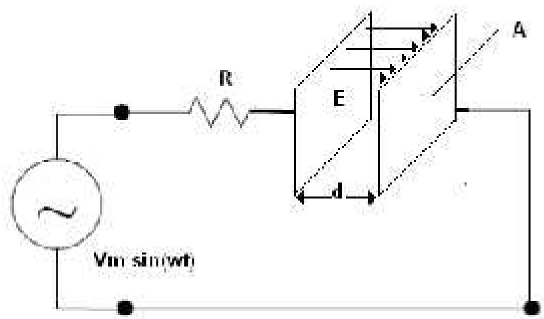
$$\int -\frac{\beta^2 H_m}{w\varepsilon_0}\sin(wt-\beta z)\,dt\,a_y = B_y$$

$$\frac{\beta^2 H_m}{w^2\varepsilon_0}\cos(wt-\beta z)\ a_y = B_y$$ (1 mark)

Note that $\frac{\beta^2}{w^2} = \frac{1}{\mu_0\varepsilon_0}$ (1 mark)

$$\mu_0 H_m\, cos(wt-\beta z)\ a_y = B_y$$ (1 mark)

(c) (i)



(2 marks)

Consider a parallel-plate capacitor in which each side or plate has an area of $A\ m^2$ and the separation between the plates is small compared to the dimensions of the plates, so that over the whole area A, a uniform electric field intensity vector $\bar{E}$ may be assumed to exist.

Let $+q$ coulomb be the total charge on one plate (and therefore automatically $-q$ is the total charge on the other plate) and v volts the potential difference between the plates. Then capacitance is defined thus

$$C = \frac{q}{v} \text{ or } v = \frac{q}{c}$$ (1 mark)

If an additional charge $(+dq)$ is placed on the positive plate, then, the work done in moving this charge is by definition

$$Voltage = \frac{work\ done}{charge}$$ (1 mark)

Therefore $work\ done = voltage\ \times charge$

Let us assume that initially both $q\ and\ v$ are zero, and finally they acquire +Q and

V values. Then the work done in this process is given by

$$Total\ work\ done = \int_0^{+Q} v dq = \int_0^{+Q} \frac{q}{c} dq = \frac{Q^2}{2C}$$ (1 mark)

If δ coulomb/m² is the uniform surface charge density on the plate, then

$$Q = \delta A$$ (1 mark)

Also $E = \frac{\delta}{\varepsilon}$ in the dielectric between the plates and capacitance then becomes

$$C = \frac{q}{v} = \frac{\delta A}{v} = \frac{\varepsilon E A}{v} = \frac{\varepsilon \frac{V}{d} A}{v} = \frac{\varepsilon A}{d}$$ (1 mark)

Using these results and noting that the volume of the dielectric between the plates is $A\,d\,m^3$. Energy stored per unit volume in the dielectric is given by

$$= \frac{work\ done}{volume}$$ (1 mark)

$$= \frac{\frac{Q^2}{2C}}{Ad} = \frac{(\delta A)^2}{2CAd} = \frac{\delta^2 A^2}{2\frac{\varepsilon A}{d} Ad} = \frac{1}{2}\varepsilon \bar{E}^2$$ (1 mark)

This is the energy density in the electric field.

(ii)