Solution Question One

(a) (i) $\Gamma_L = \frac{Z_L - Z_0}{Z_0 + Z_L} = \frac{50 - j100 - 50}{50 + 50 - j100} = \frac{-j100}{100 - j100} = 0.5 - 0.5j$

$= 0.707\angle -45^o$ (2 marks)

(ii) From $V(z) = V_0^{+} e^{-j\beta z} + V_0^{-} e^{j\beta z}$

Let $z = -l$

$V(-l) = V_0^{+} e^{j\beta z} + V_0^{-} e^{-j\beta z}$

$\Gamma(-l) = \frac{V_0^{-} e^{-j\beta z}}{V_0^{+} e^{j\beta z}}$

$\Gamma(-l) = \Gamma(0) e^{-2j\beta z}$ (2 marks)

Noting that $l = \lambda$ and $\beta l = 2\pi/\lambda \times \lambda$

$\Gamma(0.7\lambda) = \Gamma(0) e^{-j2\pi}$

$= \Gamma(0) 1\angle 0$

$= 0.707\angle -45^o$

$= 0.447\angle -170.565^o$ (2 marks)

(iii) Voltage along the line = $V(z) = V_0^{+} e^{-j\beta z} + V_0^{-} e^{j\beta z}$ Equation 1.3

$= V_0^{+} e^{-j\beta z} + \Gamma_L V_0^{+} e^{j\beta z}$

Alternatively, voltage along the line can also be determined thus:

$V(z) = \frac{Z_{in}}{Z_{in} + Z_g} V_g = V_0^{+} e^{-j\beta z} + \Gamma_L V_0^{+} e^{j\beta z}$ Equation 1.4 (2 marks)

So we have to determine Z_{in}

$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right] = Z_0 \left[\frac{100 - j50 + j50\tan(2\pi)}{50 + j(100 - j50)\tan(2\pi)}\right]$ (2 marks)

$Z_{in} = 50.0 - j100\Omega = 111.8\angle -63.435^o$ (1 mark)

Note that $l = \lambda$ and not $(-\lambda)$.

Determining the LHS of equation 1.4:

$LHS = \frac{50.0 - j100\Omega}{50.0 - j100\Omega + 50} \times 15\ V = 10.4765\angle -24.775^0\ V$ (1 mark)

$RHS = V_0^{+}(e^{-j\beta z} + \Gamma_L e^{j\beta z})$ Taking $Z = (-\lambda)$

$= V_0^{+}(e^{j1.4\pi} + \Gamma_L e^{-j1.4\pi})$

$= V_0^{+}[(\cos(2\pi) + j\sin(2\pi)) + 0.707\angle -45^o(\cos(2\pi) - j\sin(2\pi))]$

(1 mark)

$= V_0^{+} 1.4142\angle -8.13^o$ (1 mark)

$LHS = RHS$

$10.4765\angle -24.775^0 = V_0^{+} 1.4142\angle -8.13^o$ (1 mark)

$V_0^{+} = \frac{10.4765\angle -24.775^0}{1.4142\angle -8.13^o} = 7.408\angle -16.645^o$ 1 mark)

(iv) Maximum line voltage

$$VSWR = \frac{|V_{max}|}{|V_{min}|} = \frac{V_0^+ + V_0^-}{V_0^+ - V_0^-}$$

$$\frac{|V_{max}|}{|V_{min}|} = \frac{V_0^+ + \Gamma_L V_0^+}{V_0^+ - \Gamma_L V_0^+}$$

$$|V_{max}| = \frac{V_0^+ + \Gamma_L V_0^+}{V_0^+ - \Gamma_L V_0^+} \times (V_0^+ - \Gamma_L V_0^+)$$

$$|V_{max}| = 7.408(1 + 0.707) = 12.6462\ V$$ (2 marks)

(v) Minimum line voltage

$$|V_{min}| = 7.408(1 - 0.707) = 2.17\ V$$ (1 mark)

(vi) VSWR $= \frac{|V_{max}|}{|V_{min}|} = \frac{12.6462}{2.17} = 5.828$ (1 mark)

(b) (i) Challenge

A pair of lands on a PCB of length l will have a time delay in transmitting a digital pulse from one end to another of $T = \frac{l}{v}\ sec.$ This delay is becoming an increasingly important factor in the design of high speed digital devices, especially the performance of digital decoding circuits. (2 marks)

Solution

Using transmission-line model, this problem is minimized by ensuring circuit elements such as copper lands are $\lambda/20$ in length or shorter, thereby minimizing time delays in transmitting digital signals. (1 mark)

(ii) Challenge

As we go higher in frequency, capacitor values get ever smaller (into Pico and Fermto levels) and inductance values get bigger and bigger. Fabricating capacitors and inductors to be used at very high frequencies using traditional components introduces enormous practical challenges. (1 mark)

Solution

Using transmission-line model, sections of transmission lines on a PCB can be designed and constructed having any capacitive or inductive properties at the desired high frequencies with ease, and even exhibit better performance than traditional capacitors or inductors. (1 mark)

Challenge

(iii) Antennas have an input impedance Z_{out}. The antenna must inevitably be connected to the source with a transmission line section for optimal matching. If the input impedance to the antenna is different from the characteristic impedance of the transmission line, power will be reflected back to the source and not radiated out as expected. (2 marks)

Solution

Using transmission-line theory, we can provide a match between the transmission line and the antenna by designing and then inserting a $\lambda/4$ transformer between them, and in this way all of the incident energy is radiated out. (1 mark)

(iv) Challenge

Parallel transmission lines are used to connect sources to loads. The optimum transfer of power between the source and the load is determined by the reactive properties of the transmission line. Without proper matching between sources and loads, energy is reflected to and from the source with disastrous consequences to the line itself and connected equipment. (1 mark)

Solution

Transmission line theory enables us to develop a distributed model of the transmission line that allows for the determination of the characteristic impedance of the transmission line, thereby ensuring matching between sources and loads and the achievement of maximum power transfer. (1 mark)

COMPILED BY ENGINEER LUCHE VIRUS