UNIVERSITY OF NAIROBI

SECOND SEMESTER EXAMINATIONS 2015/2016

FIRST YEAR EXAMINATIONS FOR THE DEGREES OF
BACHELOR OF SCIENCE (GEOSPATIAL ENGINEERING),
BACHELOR OF SCIENCE (CIVIL ENGINEERING),
BACHELOR OF SCIENCE IN BIOSYSTEMS ENGINEERING
BACHELOR OF SCIENCE (MECHANICAL ENGINEERING),
BACHELOR OF SCIENCE (ELECTRICAL AND ELECTRONIC ENGINEERING).

FEB 112, FCE 162, FGE 172, FME 172, FEE 122: PURE MATHS IB

DATE: 23[RD] **MAY, 2016** **TIME: 9.00 A.M. – 11.00 A.M.**

Instructions: Answer question **One** and any other **Two** questions.

Question One (30 marks)

(a) Find the equations for the lines that are tangent and normal to the curve $y = \cos^2 x$ at $\left(\frac{\pi}{3}, \frac{1}{4}\right)$.

(6 marks)

(b) Show that the curve $y = x^3 - 8$ has no maximum or minimum value.

(4 marks)

(c) Let the interval $[0, 2]$ be partitioned into 8 sub intervals of equal length. Evaluate $L(f, P_8)$ and $U(f, P_8)$ for $f(x) = x$.

(6 marks)

(d) Evaluate $\int_0^1 \frac{d}{dx}\left[e^{\arctan x}\right] dx$

(5 marks)

(e) Find the volume of the solid obtained by the region bounded by $y = x^3$, $y = 8$ and $x = 0$ about the $y-$axis.

y=0

y=0

$(y^{\frac{1}{3}})^2$

24

(5 marks)

(f) Transform the polar equation $r = \frac{2}{\sqrt{\cos^2\theta + 4\sin^2\theta}}$ to the rectangular coordinates, and identify the curve represented.

(4 marks)

Question Two (20 marks)

(a) Show that $\frac{dv}{dx} = 4x\sec^3(x^2) - 2x\sec(x^2)$ where $v = \sec(x^2)\tan(x^2)$.

(6 marks)

(b) Express $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{\pi}{n}\sin\left(\frac{\pi}{n}\right)$ as a definite integral.

(10 marks)

(c) Find the average value of $f(t) = 1 + \sin t$ over $[-\pi, \pi]$.

(4 marks)

Question Three (20 marks)

(a) Sketch the graph of $f(x) = \frac{x^2-1}{x^2-4}$.

(9 marks)

(b) The velocity v of a piston is related to the angular velocity ω of the crank by the equation

$$v = \omega r\left\{\sin\theta + \frac{r}{2l}\sin 2\theta\right\}$$

where r = length of the crank, and l = length of the connecting rod. Find the first positive value of θ in degrees for which v is a maximum, for the case when $l = 4r$.

(6 marks)

(c) By about what percentage will the edge length of an ice cube decrease if the cube looses 6% of its volume.

(5 marks)

$4 = \frac{16}{4}$

$16 =$

$(x^2-4)(x^2-4)$

$x^4 - 4x^2 - 4x^2 + 16$

$x^4 - 8x^2 + 16$

$4(y^2-1)$

$4((y+1)(y-1))$

Question Four (20 marks)

(a) Evaluate the following integrals

(i)

$$\int_0^{\pi/3} \frac{dx}{1-\sin x}$$

(6 marks)

(ii)

$$\int_0^1 \tan^{-1}(x).$$

(6 marks)

(b) Express the rational function

$$f(x) = \frac{x^4 - 2x^2 + 4x + 1}{x^3 - x^2 - x + 1}$$

in the form $P(x) + \frac{R(x)}{Q(x)}$. Hence or otherwise find $\int f(x)\,dx$.

(8 marks)

Question Five (20 marks)

(a) Find the volume of the solid obtained by rotating the region bounded by $y = x - x^2$ and $y = 0$ about the line $x = 2$.

(10 marks)

(b) A hawk flying at $15m/s$ at an altitude of $180m$ accidentally drops its prey. The parabolic trajectory of the falling prey is described by the equation

$$y = 180 - \frac{x^2}{45}.$$

Calculate the distance traveled by the prey from the time it is dropped until the time it hits the ground.

$\tan 45 = 1$

(10 marks)

$\sqrt{x^2+1}$ x 1

$0.5 = \tan(30)$

$x = \tan\theta$

$\tan^{-1}(x) = \tan^{-1}(30)$

$y=0$

$4\sqrt{\frac{9}{8}}$ / 1

$2\frac{1}{4}$

$-\int + \operatorname{cosec} x$

$\int -\frac{1}{u}\,du$

$2\pi x f(x)$