UNIVERSITY OF NAIROBI

SECOND SEMESTER EXAMINATIONS – 2012/2013

FIRST YEAR EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE IN ENGINEERING

FCE 162/FEB 112/FGE 172 : PURE MATHEMATICS IB

DATE: MAY 9, 2013 **TIME: 8.30AM – 10.30AM**

INSTRUCTIONS:

Answer question **ONE** and any other **TWO** questions.

QUESTION ONE (Compulsory – 30 marks)

a) Evaluate $\int \cos 7x \cos 3x \, dx$. (3 marks)

b) Show that $f(x) = 8x - \sin 2x + 8\cos x$ is an increasing function. (3 marks)

c) Find the equation of the normal to the curve defined implicity by $2x^2 + 3y^2 - 3x + 2y = 0$ at the point $\left(1, \frac{1}{3}\right)$ (5 marks)

d) Find the polar equation for the circle $x^2 + (y-3)^2 = 9$. (3 marks)

e) Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$. (4 marks)

f) Find the length of an arc of the cycloid $x = \theta - \sin\theta$, $y = 1 - \cos\theta$ between $\theta = 0$ and $\theta = 2\pi$. (5 marks)

g) Find the equation of the curve passing through the point (3, 2) and having slope $5x^2 - x + 1$ at every point (x,y). (4 marks)

h) Evaluate $\int \frac{\sec^2 x}{(1-4\tan^2 x)^{1/2}} dx$ (3 marks)

QUESTION TWO (Optional – 20 marks)

a) i) The area enclosed by the curve $y=x^2+1$ and the line $y=3-x$ is rotated completely about the x-axis to generate a solid. Find the volume of the solid. (4 marks)

ii) The area enclosed by the curve $y=4x-x^2$ and the line $y = 3$ is rotated completely about the line $y = 3$. Find the volume of the solid generated. (4 marks)

b) A right circular cone of height h is inscribed in a sphere of radius R. Show that the volume v of the cone is given by

$$v=\frac{\pi}{3}\left(2Rh^2-h^3\right).$$

Hence, find the dimensions of the cone that give the maximum value of v. (12 marks)

QUESTION THREE (optional - 20 marks)

a) Evaluate $\int_0^{\pi/2} x^3 \cos 2x\,dx$ using Tabular integrarion. (6 marks)

b) Evaluate $\int \frac{x^2+2}{x(x+2)(x-1)}\,dx$ (7 marks)

c) A spherical raindrop is formed by condensation in an interval of 40 seconds, its volume increasing at a constant rate from 0.032 mm^3 to 0.256 mm^3. Find the rate at which the surface areas of the raindrop is increasing when its radius is 0.5 mm. (7 marks)

QUESTION FOUR (Optional – 20 marks)

a) Find the co-ordinates of the local extreme of $y=x^3e^{-6x}$ and distinguish between them. (8 marks)

b) Sketch the graph of $y=f(x)=\frac{x^2}{x-2}$. (7 marks)

c) Evaluate $\int \frac{dx}{x^{\frac{1}{2}}-x^{\frac{1}{4}}}$. (5 marks)

QUESTION FIVE (Optional – 20 marks)

a) Evaluate

i) $\int \sin^4 x \cos^7 x \, dx.$ (3 marks)

ii) $\int 4\sec^2 x e^{\tan x} \, dx.$ (2 marks)

b) Obtain the first five (5) terms of the Taylors series for Ln x about x = 2 and use it to approximate the value of Ln 2.10, given that $\text{Ln}\,2 \simeq 0.6931$. (6 marks)

c) A particle P moves in a straight line OX so that at time t, its velocity v in the direction of increasing x is $wt \sin 2wt$ where w is a constant. Given that P starts from rest at 0 when t = 0, find:

i) The acceleration of P when $t = \dfrac{\pi}{2w}$. (3 marks)

ii) The distance OP when P comes momentarily to rest for the first time after leaving O. (6 marks)

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