UNIVERSITY OF NAIROBI

SECOND SEMESTER EXAMINATIONS 2011/2012

FIRST YEAR EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE IN ENGINEERING

FEE 122: ELECTRICAL ENGINEERING
FME 172: MECHANICAL ENGINEERING

PURE MATHEMTICS B

DATE: MAY 24, 2012 TIME: 8.30 A.M. – 10. 30 A.M.

QUESTION ONE – COMPULSORY (30 MARKS)

(a) Evaluate the following definite integrals using the fundamental theorem of calculus.

(i) $\int_1^2 \frac{2x\ln(x^2+4)}{x^2+4}\,dx$ (2 Marks)

(ii) $\int_0^1 e^{4x}\sin 3x\,dx$ (2 Marks)

(iii) $\int_0^1 \frac{3^x\,dx}{\sqrt{25-9^x}}$ (2 Marks)

(b) Find the intervals of convexity and the points of inflection for $f(x)=\sqrt[3]{x^2}\,(x-5)$ (3 Marks)

(c) Show that $\int \frac{dx}{(1+x)^{1/2}(1-x)^{3/2}} = \sqrt{\frac{1+x}{1-x}} + c$ (3 Marks)

(d) Compute the area of the region bounded by the curves: $y^2 = x$; $y \geq 0$, $x = 1$ and $x = 4$. (4 Marks)

(e) Verify the formula for the volume of a cone of radius r and height h. (3 Marks)

(f) Find the arc length of the parabola $y = x^2/2$ between the points $O(0,0)$ and $A\left(\sqrt{3}, 3/2\right)$. (4 Marks)

(g) Find the shortest distance from the point $P(1,0)$ to the parabola $x = y^2$. (4 Marks)

(h) Among all pairs of positive real numbers U and V whose sum is 10, which give the greatest product UV? (3 Marks)

QUESTION TWO (20 MARKS)

(a) An open box is to be constructed with a square base and is required to have a volume of 48 cubic inches. The bottom of the box costs 3 cents per square inch, whereas the side costs 2 cents per square inch. Find the dimensions that will minimize the cost of the box. (6 Marks)

(b) Given $f(x) = x - 1/x$, find the intervals of monotonicity, extrema, intervals of convexity, points of inflection, asymptotes if any and sketch the graph of $f(x)$. (10 Marks)

(c) Find the function $f(x)$ such that $f'(x)$ is a constant, $f(0) = f'(0)$ and $f(2) = f'(2)$. (4 Marks)

QUESTION THREE (20 MARKS)

(a) Evaluate the following integrals:

(i) $\int 3^{5x^2} \cdot x \, dx$ (3 Marks)

(ii) $\int \dfrac{x \, dx}{(x-1)(x+1)^2}$ (4 Marks)

(iii) $\int \dfrac{dx}{\sin x}$ (3 Marks)

(b) Express the function $f(x)=\dfrac{x^4+x}{x^2-3x+2}$ in the form in the form $P(x)+\dfrac{R(x)}{Q(x)}$, where the degree of $R(x)$ is less than that of $Q(x)$. Hence evaluate

$$\int \frac{x^4+x}{x^2-3x+2}\,dx$$

(6 Marks)

(c) Find the equation of the curve passing through the point $(2,3)$ and having slope $3x^2-2x+5$ at each point (x,y). (4 Marks)

QUESTION FOUR (20 MARKS)

(a) Find the area of the region bounded by the parabola $y=x^2-1$ and $y=-(x^2-1)$ (3 Marks)

(b) Find the area enclosed by one loop of $r=\sin 4\theta$. (5 Marks)

(c) The part of the circle $x^2+y^2=r^2$ in the first quadrant is rotated about the x-axis. Find the area of the resulting surface of revolution. (6 Marks)

(d) Two bodies move from the same point along a straight line. The first body moves with velocity $v=(3t^2-6t)$ m/s, the second with velocity $v=(10t+20)$ m/s. At what instant and at what distance from the initial point will they meet? (6 Marks)

QUESTION FIVE (20 MARKS)

(a) Find the volume of the solid formed by the rotation of the circle $x^2+(y-b)^2=a^2\ (b\geq a)$ about the x-axis. (6 Marks)

(b) Find the length of the curve given by parametric equations:

$$\begin{cases} x=a(2\cos t-\cos 2t) \\ y=a(2\sin t-\sin 2t) \end{cases} \quad 0\leq t\leq 2\pi$$

(6 Marks)

(c) Consider the region R bounded above by the parabola $y=x^2$, below the x-axis and lying between $x=0$ and $x=1$.

Find the volume of the solid obtained by revolving R about;

(i) Horizontal line $y=-1$

(ii) Vertical line $x=-2$

(8 Marks)

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