



# UNIVERSITY OF NAIROBI

SECOND SEMESTER EXAMINATIONS 2009/2010

FIRST YEAR EXAMINATIONS FOR THE DEGREES OF BACHELOR OF SCIENCE IN:

- FCE 162: CIVIL ENGINEERING
- FEB 112: ENVIRONMENTAL AND BIOSYSTEMS ENGINEERING
- FEE 122: ELECTRICAL AND ELECTRONICS ENGINEERING JJ
- FME 172: MECHANICAL ENGINEERING
- FGS 172: GEOSPATIAL ENGINEERING

## PURE MATHEMATICS IB

DATE: JULY 22, 2010

TIME: 8.30-10.30AM

**Instructions:** ATTEMPT QUESTION ONE AND ANY OTHER TWO QUESTIONS

### Question One (30 Marks)

$$\delta y = f'(x) \cdot \delta x = 1 \times 0.02 = 0.02$$

- a) Given that  $y = 4x^2 - x$ , determine the approximate change in  $y$  if  $x$  changes from 1 to 1.02.  $\frac{dy}{dx} = (8x - 1) = 7$  [4 marks]
- b) Express:
- i)  $(2, -5)$ ,
  - ii)  $(-4, 3)$
  - iii)  $(-5, -12)$  in polar coordinates [6 marks]
- c) Evaluate the following:
- i)  $\int \cos(3x + 7) dx$  [2 marks]
  - ii)  $\int \frac{4}{(5x - 3)} dx$  [2 marks]
- d) Determine the equation of the tangent and normal to the curve  $y = \frac{x^3}{5}$  at the point  $(1, 2)$ . [4 marks]
- e) The luminous intensity  $i$  candelas of a lamp at varying voltage  $V$  is given by  $i = 4 \times 10^{-4} V^2$ . Determine the voltage at which the light is increasing at a rate of 0.6 candelas per volt.  $i = 4 \times 10^{-4} V^2$   $\frac{di}{dV} = 0.6$  [6 marks]

- f) Find the area between the curves  $y=x^2+5x$  and  $y=3-x^2$  between  $x=-2$  and  $x=0$  [6 marks]

**Question Two [20 Marks]**

- a) Determine the coordinates of the maximum and minimum values of the graph  $y = \frac{x^3}{3} - \frac{x^2}{2} - 6x + \frac{5}{3}$  and distinguish between them. Sketch the graph. [10 marks]
- b) Find the points on the centroid  $r = 1 + \cos \alpha$  at which the tangent is parallel to the initial line. [7 marks]
- c) Convert  $2x - 5x^3 = 1 + xy$  into polar coordinates. [3 marks]

**Question Three [20 Marks]**

- a) Determine the height and radius of a cylinder of volume  $200\text{cm}^3$  which has the least surface area. [8 marks]
- b) Compute the following integrals using any suitable method.
- i)  $\int_0^1 x e^x dx$  [3 marks]
- ii)  $\int \sin^{-1} x dx$  [3 marks]
- iii)  $\int_0^2 x^2 \sqrt{x^3 + 8} dx$  [3 marks]
- iv)  $\int \sin^4 x dx$  [3 marks]

**Question Four [20 Marks]**

- a) A long thin bar is made of material whose density varies along the length of the bar. Suppose the mass density is  $\rho(x) = Kx (\text{g/cm}^3)$  where  $k > 0$  is a constant and  $L$  is the length of the bar.
- i) Find the total mass of the bar. [4 marks]
- ii) Find the average mass density of the bar. [4 marks]
- b) Find the volume, if the area bounded by  $y=x^3+1$ , the x-axis and the lines  $x=0$ ,  $x=3$  is rotated around the x-axis. [7 marks]

- c) Find the area between the curve whose equation is  $y=2x+\sin x+e^x$ , the x-axis and the lines  $x=0, x=\frac{\pi}{2}$  [5 marks]

\* Question Five [20 Marks]

- a) A cup is made by rotating the area between  $y=2x^2$ , and  $y=x+1$  with  $x>0$  around the x-axis. Find the volume of the material needed to make the cup. [7 marks]
- b) A particle moves along a straight line and its retardation is  $10 e^{2x} \text{ m/s}^2$ , when its displacement is  $x$  metres from a fixed point O. Given that the initial velocity is  $10 \text{ m/s}$ , find the value of  $x$  when the particle comes to rest. [7 marks]
- c) Find the centroid of the area bounded by  $y=x^3, x=2$  and the x-axis. [6 marks]

$$V = \int_a^b \pi y^2 dx \text{ x axis}$$

$$V = \int_a^b \pi x^2 dy \text{ y axis}$$

$$\bar{x} = \frac{\int_a^b xy \cdot dx}{\int_a^b y^2 dx}$$

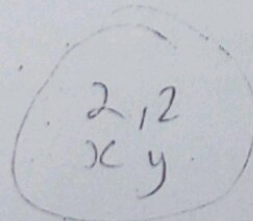
$$\bar{y} = \frac{\int_a^b y^2 dx}{\int_a^b y dx}$$

$$\frac{dy}{dx} = \frac{\Delta y}{\Delta x}$$

$$\Delta y = \frac{dy}{dx} \cdot \Delta x$$

$$\frac{3x^2}{5} \cdot m_2 = -1$$

$$3x^2 m_2 = -5$$



$$2 \cdot \left(\frac{\pi}{2}\right)$$

$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$r = \sqrt{x^2 + y^2}$$

$$\sqrt{2^2 + 2^2}$$

$$\theta = \tan^{-1} \frac{y}{x}$$

$$\tan \theta$$

$$\theta = \theta$$