UNIVERSITY OF NAIROBI

SECOND SEMESTER EXAMINATIONS 2011/2012

FIRST YEAR EXAMINATIONS FOR THE DEGREES OF BACHELOR OF SCIENCE IN ENGINEERING

FEB 114: ENVIRONMENTAL & BIOSYSTEMS ENGINEERING
FEE 112: ELECTRICAL & ELECTRONIC ENGINEERING
FME 174: MECHANICAL ENGINEERING
FGE 174: GEOSPATIAL ENGINEERING
FCE 164: CIVIL ENGINEERING

APPLIED MATHEMATICS 1B

DATE: MAY 29, 2012 **TIME: 8.30 A.M. – 10.30 A.M.**

INSTRUCTIONS

Answer Question **ONE** and any other **TWO** Questions. use $g=9.81\ m/s^2$

QUESTION ONE (30 marks)

a) A bullet of mass 10g travelling at 300m/s hits a lump of wood of mass 590g which is lying on a rough table. If the bullet becomes embedded in the wood and they travel together for 3s before coming to rest, find the speed of the wood immediately after impact and the coefficient of friction. (5 marks)

b) Water is pumped, at the rate of $12m^3$ per minute from a reservoir to a height of 25m, where it is ejected from a pipe whose cross-sectional area is $50cm^2$. Calculate the power needed. (mass of $1m^3$ of water is 1000kg). (4 marks)

c) For the frequency table below, it is given that the sum of the frequencies is 458 and the median is 46. Calculate the values of f_1 and f_2.

Class	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Freq.	24	f_2	67	130	91	50	f_1

(5 marks)

$46 = 39.5 + \left[\frac{229.5-(91+f_2)}{130}\right]10$ $f_2 = 54$

$24t^4(2t^2-5t+1)-20(2t^2-5t+1)$

$48t^5-120t^3+120t^2-40t^3+100t-100$

d) The velocities of a ball before and after being struck by a bat are: $v_1 = [-2, 5, 0]$ m/s and $v_2 = [4, -3, 0]$ m/s respectively. If the ball weights 0.3kg and contact lasts for $\frac{1}{30}$s, calculate the impulse imparted by the bat on the ball and the magnitude of the average force between the bat and the ball. (4 marks)

e) A body performs simple harmonic motion. Its displacement is at its maximum value of 0.40m when t=0. It first reaches a point 0.20m from its equilibrium point after 3.0s.

i) Find the period of motion

ii) Find the body's displacement when its speed is 0.05 m/s. (6 marks)

f) When a die is rolled and a coin (with heads and tails) tossed, find the probability of obtaining:

i) Tails and an even number

ii) A number greater than 3

iii) Heads or a 3 given that the number rolled on the die was odd. (6 marks)

QUESTION TWO

a) Define simple harmonic motion. (2 marks)

b) A particle is executing simple harmonic motion in a straight line with period T and amplitude a. The particle has velocity v when at a distance x from the equilibrium position. Show that:

$$vT = 2\pi\sqrt{a^2 - x^2}$$ (4 marks)

Given that the particle has velocities 3m/s and 2m/s when its distances from the equilibrium position are 2m and 3m respectively, find the amplitude and the period. (4 marks)

b) A flywheel has a constant angular deceleration of 2.0 rad/s^2.

i) Find the angle through which the flywheel turns as it comes to rest from an angular speed of 220 rad/s.

ii) Find the time required for the flywheel to come to rest. (5 marks)

$v = u + at$

$a = \frac{v-u}{t}$

$at = v - u$

$v = u + at$

c) A simple harmonic oscillator has a period of 0.0318s and has a mass of 0.185 kg that moves with an amplitude of 0.0650m.

i) What is the total energy of the system?

ii) When the mass is displaced 0.0340m from equilibrium, what is the kinetic energy? (5 marks)

QUESTION THREE

a) Show that the work-done by a variable force is equal to the change in kinetic energy. (6 marks)

b) A particle of mass 4 units moves in a force field, depending on time t, given by:

$$\bar{F} = (24t^2 - 20)\hat{i} + 16t\hat{j} + (12t + 4)\hat{k}$$

Given that at time t=0, the particle has velocity $\bar{v}_0 = 5\hat{i} - 7\hat{j} + 9\hat{k}$ and is situated at $\bar{r}_0 = 2\hat{i} + 8\hat{j} - 5\hat{k}$.

i) Find the velocity and position of the particle at any time t.

ii) Calculate the work-done by the force in moving the particle from t=1 to t=4. (8 marks)

c) When a car of mass 1200kg is driving up a hill inclined at α to the horizontal, the engine is working at 32kW and the maximum speed is 25 m/s. Given that $\sin\alpha = \frac{1}{16}$ and assuming that the car can be modelled as a particle, find the resistance due to friction. (6 marks)

QUESTION FOUR

a) Define the angular momentum of a particle. (2 marks)

The rotating parts of an electric motor have a moment of inertia of 12kgm^2. When running at 1200 rev/min it is connected by means of a disk clutch to another shaft at rest, which has a moment of inertia of 24 kgm^2. Find the common speed of rotation immediately after slip has ceased. If the electric motor exerts a constant torque of 160 Nm, find the time taken for the two shafts together to regain the initial speed of 1200 rev/min. (4 marks)

b) Two particles A and B, of masses M and m respectively lie at rest inside a smooth horizontal tube. A and B are projected towards each other with speeds 5 m/s and 2 m/s respectively.

After impact A continues to move in the same direction as before at 1 m/s while B's direction of motion is reversed and its speed becomes 3 m/s. Calculate the ratio M/m. if the loss in K.E is 100J, calculate the values of M and m. (8 marks)

c) A particle of mass m moving in a straight line with speed u receives an impulse of magnitude I in the direction of its motion.

Show that the increase in its kinetic energy is given by:

$$\frac{I(I+2mu)}{2m}$$ (6 marks)

QUESTION FIVE

If you choose this question (Q5), answer either part I OR part II.

PART I

a) A discrete random variable x has the Poisson Distribution: $x \sim p(\lambda)$ such that:

$$P(x=r)=\frac{\lambda^2 e^{-\lambda}}{r!} \text{ where } r=0,1,2.....$$

Show that the mean of x is equal to λ. (4 marks)

Given $x \sim P(1.5)$, find

i) $P(x=0)$
ii) $P(x=1 \text{ or } 3)$
iii) $P(x<5)$

(6 marks)

b) A discrete random variable x is normally distributed with mean $\mu=300$ and variance $\sigma^2=25$: $x \sim N(300, 25)$. Find:

i) $P(x>305)$
ii) $P(x \leq 312)$
iii) $P(288<x \leq 310)$.

(5 marks)

c) The probability that a day will be fine is 0.4. Find the expected number of fine days in a week and the standard deviation. (5 marks)