JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICAL & ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR THE MASTERS DEGREE

IN APPLIED MATHEMATICS

1ST YEAR 1ST SEMESTER 2013/2014 ACADEMIC YEAR

CENTRE: MAIN SCHOOL BASED

COURSE CODE: SMA 840

COURSE TITLE: METHODS OF APPLIED MATHEMATICS

EXAM VENUE: **STREAM: (MSc.)**

DATE: 29/4/2014 **EXAM SESSION: 9.00 – 12.00 NOON**

TIME: 3 HOURS

Instructions:

1. **Answer question 1 (Compulsory)and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

Question1[20 marks]

(a)Solve the system of ordinary differential equations $\frac{dx}{dt}+\frac{dy}{dt}+y+x=1,\ \frac{dy}{dt}-2x-y=0:\ x(0)=0\,,y(0)=1$ [14 marks]

(b) An electrical circuit gives rise to the system

$$L\,I_1' \ + RI_1+\frac{q}{C}=E_0(t)\,.$$

$$L\,I_2' \ + RI_2-\frac{q}{C}=0\,.$$

$$q'-I_1 \ +I_2=0$$

If initially the charge q is zero, and the currents $I_1(0)=I_2(0)=\frac{E_0}{2R}$, solve the system. [6 marks]

Question2[20 marks]

(a)Evaluate the integral $\int_H \frac{dz}{\left(z^2-8z+25\right)^4}$ where H is a semi circle in the lower half plane big enough to contain all poles of this integrand with negative imaginary part. [16 marks]

(b) Obtain solution to the initial value problem

$xy''+(1-x)y'+4y=0;$ satisfying $y(0)=e^{-300},\ y'(0)=e^{12}$ [4 marks]

Question3 [20 marks]

(a)Show that $\underset{\sim}{v}_1=[-4,5,7]^t,\ \underset{\sim}{v}_2=[-3,4,2]^t$ are eigenvectors of

the matrix $A=\begin{pmatrix}1 & 1 & 1\\ 2 & 1 & -1\\ -8 & -5 & -3\end{pmatrix}$.

(b)Find e^{At}, the exponential matrix of A

(c)Verify that $\left[e^{At}\right]_{t=0}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$

Question4 [20 marks]

Given the system of first order ordinary linear differential equations $\dot{\underset{\sim}{X}} = A\underset{\sim}{X} + \underline{F}(t)$ where

$$A = \begin{bmatrix} 1 & -1 \\ 2 & -2 \end{bmatrix}, \underline{F}(t) = \begin{bmatrix} e^{t} \\ e^{-t} \end{bmatrix}$$

(a)Find the fundamental matrix $\Phi(t)$ of the system. [10 marks]

(b) (i)Evaluate the matrix $\left[\Phi(t)\Phi^{-1}(0)\right]_{(t=0)}$.(ii) Deduce $\underset{\sim}{X}$ the general solution of the system $\dot{\underset{\sim}{X}} = A\underset{\sim}{X} + \underline{F}(t)$. [10 marks]

Question5 [20 marks]

An elliptic partial differential equation is defined in the region $\Omega = \{0 < x < a, 0 < y < b\}$ by

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + 2\frac{\partial u}{\partial x} = f(x,y) \quad in \quad \Omega: \qquad 0 < x < a, \quad 0 < y < b \qquad \text{(i)}$$

$u = g(x,y) \qquad on \quad S; \qquad S$ is the boundary of Ω

Determine the Green's function $G(x, y; ¶, ¦)$ for the elliptic partial differential equation.

[20 marks]

LAPLACE TRANSFORMS TABLE

$\cos bt$	$\dfrac{s}{s^2+b^2}$
$\sin bt$	$\dfrac{b}{s^2+b^2}$
$e^{-at}\sin bt$	$\dfrac{b}{(s+a)^2+b^2}$
$e^{-at}\cos bt$	$\dfrac{(s+a)}{(s+a)^2+b^2}$
$e^{-at}t^n$	$\dfrac{\Gamma(n+1)}{(s+a)^{n+1}} \quad n>-1$
t^n	$\dfrac{n!}{s^{n+1}}$
$e^{-at}t^n$	$\dfrac{n!}{(s+a)^{n+1}}$
$\dfrac{dy}{dt}$	$sY-y_0 \quad ; \quad Y=L(y)$
$\dfrac{d^2y}{dt^2}$	$s^2Y-sy_0-y_0' \quad ; \quad Y=L(y)$
$J_0(t)$	$\dfrac{1}{\sqrt{s^2+1}}$
$t\sin bt$	$\dfrac{2bs}{(s^2+b^2)^2}$
$t\cos bt$	$\dfrac{s^2-b^2}{(s^2+b^2)^2}$

$$L\left[\frac{f(t)}{t}\right]=\int_t^{\infty} F(s)ds : L\{f(t)\}=F(s)$$

$$L^{-1}\left[L\{f(t)\}L\{g(t)\}\right]=\int_0^1 f(a)\,g(t-a)\,da \;,$$

$f(t)$	Laplace transform of $f(t)$
$J_0(t)$	$\frac{1}{\sqrt{s^2+1}}$
$\sin bt$	$\frac{b}{s^2+b^2}$
$L_m(x)=\frac{e^x}{m!}\frac{d^m}{dx^m}\left(x^m e^{-x}\right) \quad m=0,1,2,....$	
$e^{-at}\sin bt$	
$e^{-at}\cos bt$	$\frac{(s-1)^m}{s^{m+1}}$
$e^{-at}t^n$	$\frac{b}{(s+a)^2+b^2}$
t^n	$\frac{(s+a)}{(s+a)^2+b^2}$
$e^{-at}t^n$	$\frac{\Gamma(n+1)}{(s+a)^{n+1}} \quad n>-1$
$\frac{dy}{dt}$	$\frac{n!}{s^{n+1}}$
$\frac{d^2y}{dt^2}$	$\frac{n!}{(s+a)^{n+1}}$
	$sY-y_0 \quad Y=L(y)$
	$s^2Y-sy_0-y_0' \quad Y=L(y)$

$L\left[\frac{\partial u(x,t)}{\partial t};s\right]$	$sU(x,s)-u(x,0)$
$L\left[\frac{\partial^2 u(x,t)}{\partial t^2};s\right]$	$s^2U(x,s)-su(x,0)-u_t(x,0)$
$L\left[\frac{\partial u(x,t)}{\partial x};s\right]$	$\frac{dU(x,s)}{dx}$
$L\left[\frac{\partial^2 u(x,t)}{\partial x^2};s\right]$	$\frac{d^2U(x,s)}{dx^2}$
$L\left[\frac{\partial^2 u(x,t)}{\partial x\partial t};s\right]$	$s\frac{dU(x,s)}{dx}-\frac{du(x,0)}{dx}$

$J_0(t)$ is the Bessel function of order zero.

$L_m(x)$ is the Laguerre's function of order m

$L^{-1}\{¤(s)\}=e^{-at}L^{-}\{¤(s-a)\}$, $L\{e^{-at}f(t)\}=L\{f(t)\}_{s\to s+a}$

LAPLACE TRANSFORMS