JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY SCHOOL OF MATHEMATICAL \& ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR THE BACHELORS DEGREE
$1^{\text {ST }}$ YEAR $1^{\text {ST }}$ SEMESTER 2013/2014 ACADEMIC YEAR CENTRE: MAIN

COURSE CODE: SMA 3111
CO X URSE TITLE: MATHEMATICS 1
EXAM VENUE: AH
STREAM:

DATE: 23/4/2014
EXAM SESSION: 11.30-1.30 PM
TIME: 2 HOURS

## Instructions:

1. Answer question 1 (compulsory) and ANY other 2 questions.
2. Candidates are advised not to write on the question paper.
3. Candidates must hand in their answer booklets to the invigilator while in the examination room.

## QUESTION ONE (30 marks)

a) The following information refers to the functions $f$ and $g$ :

$$
\begin{aligned}
& g: x \rightarrow 7 x-6 \\
& f: x \rightarrow \frac{11}{2 x-7}, x \neq k
\end{aligned}
$$

Find
(i) the value of $k$, (1 marks)
(ii) $g f(x) .(4$ marks $)$
b) Describe the following sets using the list method and give the set cardinality:

$$
\left\{x \mid x=4 n, n^{2}<34, n \text { a positive integer }\right\}(5 \text { marks })
$$

c) Convert:
(i) $37.6471^{\circ}$ to $D^{o} M^{\prime} S^{\prime \prime}$ form. Round your answer to the nearest second.(3 marks)
(ii) $-330^{\circ}$ to radians. Express your answer as a multiple of $\pi$. ( 3 marks)
d) Solve the equation: $\log _{5}(x+6)=1-\log _{5}(x+4)(4$ marks $)$
e) Find the coefficient of the term $x^{6} y^{5}$ in the expansion of $(2 x-5 y)^{11}$ (4 marks)
f) Compute the mean, median and mode(s) of the following set of data:
$\{4.72,4.71,4.74,4.73,4.72,4.71,4.73,4.72\}$ ( 6 marks)

## QUESTION TWO (20 marks)

a) Consider the following sets:

$$
U=\{1,2,3, \ldots, 10\}, P=\{1,2,3,4,5\}, Q=\{2,4,6,8\}, R=\{1,3,5,7,9\}, \text { and } S=\{1,2,9,10\}
$$

Find:
(i) $(P-(Q \cap R)) \cap S$ (4 marks)
(ii) $(P \cap Q)-(P \cup Q)^{\prime}(4$ marks $)$
b) Prove the following distributive law of set operations:

$$
F \cap(G \cup H)=(F \cap G) \cup(F \cap H)(6 \text { marks })
$$

c) Draw the Venn diagram for the combination of the sets $A, B$, and $C$ :

$$
\left(A^{c} \cup B\right) \cup C^{c}(6 \text { marks })
$$

## QUESTION THREE (20 marks)

a) Solve $3(1-\cos \theta)=\sin ^{2} \theta$ on the interval $0 \leq \theta \leq 2 \pi$. ( 6 marks)
b) In the diagram, $A B$ is an arc of a circle centre $O$ and radius $r \mathrm{~cm}$, and angle $A O B=\theta$ radians. The point $x$ lies on $O B$ and $A X$ is perpendicular to $O B$

(i) Show that the area, $A \mathrm{~cm}^{2}$, of the shaded region $A X B$ is given by

$$
A=\frac{1}{2} r^{2}(\theta-\sin \theta \cos \theta)(8 \text { marks })
$$

(ii) In the case where $r=12$ and $\theta=\frac{1}{6} \pi$, find the perimeter of the shaded region $A X B$, leaving your answer in terms of $\sqrt{3}$ and $\pi$. ( 6 marks)

## QUESTION FOUR (20 marks)

a) The first three terms of an infinite geometric progression are $0.7,0.07,0.007$.
(i) Write down the common ratio of this progression. (2 marks)
(ii) Find, as a fraction, the sum to infinity of the terms of the progression. (4 marks)
(iii) Find the sum to infinity of the geometric progression $0.7-0.07+0.007-\ldots$, and hence show that $\frac{7}{11}=0 . \dot{6} \dot{3}$. $(6$ marks $)$
b) A pendulum is set swinging. Its first oscillation is through an angle of $30^{\circ}$ and each succeeding oscillation is through $95 \%$ of the angle of the one before it.
(i) After how many swings is the angle through which it swings less than $1^{0}$ ? (4 marks)
(ii) What is the total angle it has swung through at the end of its tenth oscillation? (4 marks)

## QUESTION FIVE (20 marks)

The following distribution gives the finishing times in minutes for male runners in a marathon:

| Time | $110-119$ | $120-129$ | $130-139$ | $140-149$ | $150-159$ | $160-169$ | $170-179$ | $180-189$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Runners | 5 | 7 | 12 | 20 | 16 | 10 | 7 | 3 |

a) Compute mean and standard deviation from the above data, ( 10 marks)
b) Draw the ogive curve from the above data and estimate the median time. ( 10 marks)

