

JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY SCHOOL OF MATHEMATICAL & ACTUARIAL SCIENCE UNIVERSITY EXAMINATION FOR THE BACHELORS DEGREE 2ND YEAR 1ST SEMESTER 2013/2014 ACADEMIC YEAR

CENTRE: MAIN

COURSE CODE: SMA 200 COURSE TITLE: CALCULUS II EXAM VENUE: AH STREAM: (BSc. Actuarial, Bed,B Sc) DATE: 14/4/2014 EXAM SESSION: 9.00 – 11.00 AM TIME: 2 HOURS

Instructions:

- 1. Answer question 1 (Compulsory)and ANY other 2 questions
- 2. Candidates are advised not to write on the question paper.
- **3.** Candidates must hand in their answer booklets to the invigilator while in the examination room.

QUESTION ONE (COMPULSORY) (30 marks)

- a) If $y' = f'(x) = 2x^2 + x 1$ and f(0) = 0, then determine a relation in y and x. (4 marks)
- b) Evaluate the integral $\int \sqrt{1 + \sin 2x} \, dx$ (5 marks)
- c) Verify by differentiation that the formula is correct:

$$\int \frac{1}{\sqrt{a^2 + x^2}} dx = \log \left| x + \sqrt{a^2 + x^2} \right| + C$$
 (6 marks)

d) Evaluate the improper integral

$$\int_{-\infty}^{\infty} \frac{2x}{(x^2+1)^2} dx \text{ (5 marks)}$$

e) Find the length of the curve $x = \left(\frac{y^{\frac{3}{2}}}{3}\right) - y^{\frac{1}{2}}$ from $y = 1$ to $y = 9$ (5 marks)

f) Determine whether the following series converges or diverges

$$\sum_{n=1}^{\infty} \frac{n2^2 (n+1)!}{n^2 !}$$
(5 marks)

QUESTION TWO (20 marks)

a) Evaluate the integral

$$\int \frac{x}{1-x^2+\sqrt{1-x^2}} dx$$
 (6 marks)

b) Evaluate

$$\int \frac{x^3}{\sqrt{1-x^8}} dx \tag{4 marks}$$

- c) By making the appropriate substitution for *u* in the integral below:
 - (i) Express the integral in terms of u. (3 marks)
 - (ii) Evaluate the integral as function of *x*. (3 marks)

$$\int_{1}^{3} \frac{2x-1}{(x+1)^4} dx$$

d) Evaluate the integral

$$\int_{0}^{t/4} \frac{1}{\sin_{\#} + \cos_{\#}} d_{\#} \tag{5 marks}$$

QUESTION THREE (20 marks)

a) Determine the value of the integral

$$\int_{2}^{3} \frac{1}{3 - 2x - x^2} dx$$
 (5 marks)

b) Evaluate the integral

$$\int \frac{x^3 + 4x^2 - x}{(x+2)(x+1)} dx$$
 (8 marks)

(7 marks)

c) Integrate by parts

$$\int e^{ax} \sin bx \, dx$$

QUESTION FOUR (20 marks)

- a) Find the volume of the solid generated by revolving the region bounded by the curve
- $y = 4 x^2$ and line y = 2 x about the *x*-axis. (7 marks) b) Determine the area of the surface generated by revolving the curve $y = \sqrt{2x - x^2}$, $0.5 \le x \le 1.5$ about the *x*-axis. (6 marks)
- c) Find the area of the region enclosed by the line $x + y^2 = 3$ and the curve $4x + y^2 = 0$. (7 marks)

QUESTION FIVE (20 marks)

a) Evaluate $\int_{0}^{1} e^{x^2} dx$ by Simpson's rule taking ten intervals. (5 marks) b) Find a power series for the logarithmic function

$$L(x) = \ln(1+x)^{3}$$
(6
marks)

- c) Show that the Taylor series about x = 0 for the function $f(x) = \cos x$ is $\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$ (5 marks)
- d) Evaluate the following integral

$$\int \frac{x^2 \tan^{-1} x^3}{1+x^6} dx \tag{4 marks}$$