JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

KISII LEARNING CENTRE

FIRST YEAR SEMESTER TWO EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION (ARTS)

SMA 103- LINEAR ALGEBRA I **TIME- 2HOURS**

INSTRUCTIONS: Answer question one (compulsory) and any other two questions

QUESTION ONE (30MARKS)

a) Consider the following systems of linear equations;

$x + 2y = 4$

$2x - y = 3$

$3x + y = k$

Find the value of *k* for which the system is consistent (3mks)

b) Use Gauss- Jordan Elimination method to solve the following systems of linear equations (6mks)

$x_1 + x_2 + 2x_3 = 8$

$x_1 + 2x_2 - 3x_3 = -1$

$3x_1 - 7x_2 + 4x_3 = 10$

c) Find the values of λ for which the determinant of the matrix below is equal to zero

$$\begin{bmatrix} \lambda+1 & 0 & 0 \\ 4 & \lambda & 3 \\ 2 & 8 & \lambda+5 \end{bmatrix}$$ (5mks)

d) Let $A = \begin{bmatrix} 1 & 2 & -2 \\ 2 & -3 & 10 \\ 0 & 1 & -3 \end{bmatrix}$, find the adjoint of *A* and hence determine the value of A^{-1}. (8mks)

e) i) State the Cramer's rule (2mks)

ii)Use Cramer's rule to solve the following systems of linear equations;

$$-x_1 + 2x_2 - 3x_3 = 3$$
$$2x_1 + x_3 = 0$$
$$3x_1 - 4x_2 + 4x_3 = 6$$

(6mks)

QUESTION TWO (20 MARKS)

a) i) Define the term Basis of a vector space *V* (2mks)

ii) Let $S = \{ V_1=(1,2,1), V_2 =(2,9,0), \text{ and } V_3 =(3,3,4)\}$. Show that the set S is a basis for $\mathbb{R}^3$ (5mks)

b) Show that $\langle u, v\rangle = u_1v_1 + 2u_2v_2$ where $u = (u_1, u_2)$, $v = (v_1, v_2)$ is an inner product on $\mathbb{R}^2$ (6mks)

c) i) Define the term Linear transformation (1mk)

ii) Show that $T:\mathbb{R}^3 \rightarrow \mathbb{R}^2$ defined by $T\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 \\ x_2 - x_1 \end{bmatrix}$ is a linear transformation

(6mks)

QUESTION THREE (20 MARKS)

a) i) Define the term a symmetric matrix (1mk)

ii) Prove that a symmetric matrix of order 2 is diagonalizable (4mks)

iii) State the Cayley – Hamilton's theorem and use it to verify for the matrix

$A=\begin{bmatrix} 1 & -3 \\ 2 & 5 \end{bmatrix}$ (4mks)

b)i) Show that the determinant of a second order matrix with identical rows is zero

(2mks)

ii) Consider the matrices $A=\begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$, $B=\begin{bmatrix} 1 & -4 \\ 4 & -1 \end{bmatrix}$ determine whether these matrices

Commute andhence find the commutator (4mks)

d) Use Cramer's rule to find the point of intersection of the three planes defined by;

$$x + 2y - z = 4$$
$$2x - 2y + 3z = 3$$
$$4x + 3y - 2z = 5$$

(5mks)

QUESTION FOUR (20 MARKS)

a) Calculate the area of the triangle whose vertices are A(1,0), B(2,2) and C(4,3) by use of the method of determinants (4mks)

b) Consider the vectors $u= (1,-3, 7)$ and $v= (8,-2,-2)$. Find $u.v$ and the angle between them (5mks)

c) i) Let $V=\mathbb{R}^3$ with standard operations and S={ (1,2,3), (0,1,2), (-2,0,1)}≤ $\mathbb{R}^3$. Does S Span V? (3mks)

ii) Let $V=\mathbb{R}^3$ and S={ (-4,-3,4), (1,-2,3), (6,0,0)}.Determine whether S is linearly independent. (4mks)

d) Find the basis and dimension for the solution space of the homogeneous system

$2x_1 + x_2 +3x_3 = 0$

$x_1+5x_3 = 0$

$x_2+x_3 = 0$ (4mks)

QUESTION FIVE (20 MARKS)

a) Apply the Gram-Schmidt process to construct an orthonormal basis set for $B=\{(1,1,0), (1,2,0), (0,1,2)\}$ of $\mathbb{R}^3$. (7mks)

b) Show that the transformation $T(x)=2x+1$ is not a linear transformation (3mks)

c) Find the eigen values of the matrix A= $\begin{bmatrix} 5 & 2 \\ 9 & 2 \end{bmatrix}$ (5mks)

d) Let A= $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$. Determine whether or not A is diagonalisable (5mks)