JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY SCHOOL OF MATHEMATICAL \& ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR THE BACHELORS DEGREE $1^{\text {ST }}$ YEAR $1^{\text {ST }}$ SEMESTER 2013/2014 ACADEMIC YEAR

CENTRE: MAIN

COURSE CODE: SMA 100
COURSE TITLE: BASIC MATHEMATICS

EXAM VENUE: AH STREAM: (BSc. Actuarial, Bed,B Sc)
DATE: 16/4/2014
EXAM SESSION: 2.00-4.00 PM
TIME: 2 HOURS

## Instructions:

1. Answer question 1 (Compulsory)and ANY other 2 questions
2. Candidates are advised not to write on the question paper.
3. Candidates must hand in their answer booklets to the invigilator while in the examination room.

## QUESTION ONE (30 marks)

a) A man wishes to save money by setting aside 1 shilling the first day, 2 shillings the second day, 4 shillings the third day, and so on:
(i) If he continues to double the amount set aside each day, how much must he set aside on the fifteenth day?
(ii) Assuming he does not run out of money, what is the total amount saved at the end of the 30 days?
b) Verify the identity
$\frac{\cos \theta}{1-\sin \theta}=\frac{1+\sin \theta}{\cos x}$
c) Given that $P=\{0,1,2,3\}$ and $Q=\{p, q\}$. Show that the $P \times Q \neq Q \times P$
d) Find the first six terms of the expansion of $\sqrt{\frac{1-x}{1+x}}$ in ascending powers of $x$. By taking the first three terms of the expansion and substituting $x=\frac{1}{8}$, find the value of $\sqrt{7}$, correct to six significant figures (6 marks)
e) Write $\left[\sqrt{3}\left(\cos \frac{5 \pi}{8}+i \sin \frac{5 \pi}{8}\right)\right]^{6}$ in the standard form $a+i b$
f) Solve $4^{x}-2^{x}-12=0$

## QUESTION TWO (20 marks)

a) You are interviewing for a job and receive two offers:

A: Kshs 20,000 to start, with guaranteed annual increases of $6 \%$ for the first 5 years.
$B: K s h s 22,000$ to start, with guaranteed annual increases of $3 \%$ for the first 5 years.
(i) Which offer is better if your goal is to be making as much as possible after 5 years? ( 6 marks)
(ii) Which is better if your goal is to make as much money as possible over the contract of five years? ( 6 marks)
b) The first, seventh and tenth terms of an arithmetical progression are in geometrical progression. If the third term is twice the eighth term, find the common difference and the common ration. (8 marks)

## QUESTION THREE (20 marks)

a) Solve the following equation:

$$
\sin ^{2} \theta=6(\cos (-\theta)+1) \text { for } \theta \text { where } 0 \leq \theta \leq 2 \pi \text {. ( } 6 \text { marks) }
$$

b) Eliminate $\theta$ from the equations:

$$
x=2 \sec \theta, \quad y=\cos 2 \theta \quad(4 \text { marks })
$$

c) Prove the following identity:

$$
\frac{\sin \theta+\cos \theta}{\cos \theta}-\frac{\sin \theta-\cos \theta}{\sin \theta}=\sec \theta \csc \theta \quad(5 \text { marks })
$$

d) Show that the area $A$ of an isosceles triangle whose equal sides are of length $s$ and $\theta$ is the angle between them is $A=\frac{1}{2} s^{2} \sin \theta$ (5 marks)

## QUESTION FOUR (20 marks)

a) Describe the following set using the list method and give the set cardinality

$$
T=\left\{x \mid 2 x^{2}-8 x-5=0, x, \text { a real number }\right\}(4 \text { marks })
$$

b) If $U=\{a, b, c, d, e, f, g\}, S=\{a, b, c, d, e, g\}, T=\{a, b, f, g\}, V=\{d\}$, and $W=\{a, b, d, e, g\}$. Find

$$
\left[(S \cup W) \cup T^{\prime}\right]^{\prime}(5 \mathrm{marks})
$$

c) On a standard 3-circle Venn diagram, shade the region corresponding to the set:

$$
\left(P^{\prime} \cap Q\right) \cup(P \cap R) \text { (6 marks) }
$$

d) Prove the following law of set operations

$$
(G \cup H)^{c}=G^{c} \cap H^{c}(5 \text { marks })
$$

## QUESTION FIVE (20 marks)

a) Solve the following system of equations

$$
\left\{\begin{array}{c}
2 x+3 y-z=3 \\
x-y-z=0 \\
-x+y+z=0 \\
x+y+3 z=5
\end{array}\right.
$$

Using matrices (row operations). If the system has no solution, say that it is inconsistent. (10 marks)
b) Solve the system of equations

$$
\begin{aligned}
x+2 y-z & =-3 \\
2 x-4 y+z & =-7 \\
-2 z+2 y-3 z & =4
\end{aligned}
$$

using Cramer's Rule if it is applicable. If Cramer's Rule is not applicable, say so. (10 marks)

