

JOOUST UNIVERSITY  
UNIVERSITY EXAMINATIONS 2013/2014  
SECOND YEAR FIRST SEMESTER EXAMINATION FOR THE DEGREE OF  
BACHELOR OF SCIENCE IN ACTUARIAL SCIENCES WITH IT  
(MAIN-CAMPUS)  
SAC 201: FINANCIAL MATHEMATICS I

DATE: April, 2014

TIME: 2 Hours

Instructions:

- Answer Question **ONE** and **ANY** other **TWO** questions.
  
- Show all your workings.

## QUESTION ONE

- (a) Define the following terms as used in financial mathematics
- Sinking fund.
  - Depreciation. [2 marks]
- (b) Kelly wishes to buy a car that costs \$32,998. The car dealer tells her that they can finance the car at 6.25% per year compounded monthly for 5 years. She decides to secure the loan from the dealer. How much will her monthly payments be? [4 marks]
- (c) Measure time in years from the present, and suppose that  $\delta(t) = 0.08(0.9)^t$  for all  $t$ . Find a simple expression for  $v(t)$ , and hence find the discounted present value of ksh. 250,000 due in five years' time. [5 marks]
- (d) The force of interest per unit time is,  $\delta(t)$ , where time is measured in years, equals 0.15 for all  $t$ . Find the nominal rate of interest per annum on deposits of term one month. [4 marks]
- (e) The effective rate of interest per annum on a certain building society account is at present 9%, but in two years' time it will be reduced by 2%. Find the accumulation in six years' time of an investment of ksh. 400000 in this account. [4 marks]
- (f) Assume that time is measured in years and that

$$\delta(t) = \begin{cases} 0.04, & t < 15; \\ 0.03, & t \geq 15. \end{cases}$$

Find  $v(t)$  for all  $t$ . [5 marks]

- (g) A loan is repaid with level annual payments based on an annual effective interest rate of 7%. The 8th payment consists of 789 of interest and 211 of principal. Calculate the amount of interest paid in the 18th payment. [5 marks]
- (h) A capital redemption policy with sum assured of ksh. 1000000 and term of 15 years has level annual premiums, payable in advance throughout the duration of the policy. The company issuing the policy calculated the annual premium on the basis of an interest rate of 8% with allowance for
- initial expenses of ksh. 10000 plus 10% of the first annual premium and
  - renewal expenses of 4% of the second and each subsequent annual premium.

Find the annual premium for this policy. [5 marks]

## QUESTION TWO

- (a) Derive an expression for the present value of an annuity-due. [2 marks]
- (b) A loan of Ksh 480000 is to be repaid by 20 equal annual instalments. The rate of interest for the transaction is 15% p.a. Find the amount of each annual repayment, assuming that payments are made
- in arrear. [5 marks]
  - in advance. [5 marks]
- (c) The amount of principal repaid in the 3<sup>rd</sup> payment of a 5-year loan at 9% is 300. What is the original loan value? [5 marks]
- (d) In return for a loan of ksh. 100000 a borrower agrees to repay ksh. 110000 after eight months. Find the force of interest per annum for the transaction. [3 marks]

### QUESTION THREE

- (a) Let time be measured in years and suppose that for all  $t_1 \leq t_2$

$$A(t_1, t_2) = \exp[0.09(t_2 - t_1)]$$

- i. Verify that the principle of consistency holds. [4 marks]
  - ii. Find the accumulation nineteen years later of an investment of ksh. 200000 at any time. [4 marks]
- (b) If  $\delta(t)$  and  $A(t_0, t_1)$  are continuous functions of  $t$  for  $t \geq t_0$  and the principle of consistency holds, then, for  $t_0 \leq t_1 \leq t_2$ , show that

$$A(t_1, t_2) = \exp\left(\int_{t_1}^{t_2} \delta(t) dt\right)$$

[4 marks]

- (c) Calculate the time in days for 3,600 to accumulate to 4,000 at:

- i. a simple rate of interest of 6% per annum;
- ii. a compound rate of interest of 6% per annum convertible quarterly;
- iii. a compound rate of interest of 6% per annum convertible monthly;
- iv. a compound force of interest of 6% per annum. [8 marks]

### QUESTION FOUR

- (a) A member of a pensions savings scheme invests 1,200 per annum, paid annually in advance, for 20 years from his 25th birthday. Calculate the accumulation of this investment at the 45th birthday if the rate of interest is constant at 5% per annum effective. [3 marks]
- (b) From the 45th birthday, the member increases his investment to 2,400 per annum. At the same time, the effective interest rate drops to 2% per annum. The investments continue to be made annually in advance for 20 years until the individuals 65th birthday. Calculate the accumulation of the entire investment at the 65th birthday. [3 marks]
- (c) Immediately after the 65th birthday, the scheme member uses his accumulated investment to purchase an annuity with a term of 20 years to be paid monthly in arrears. During this time the effective interest rate remains at 2% per annum. Calculate the amount of the monthly payment of the annuity. [5 marks]
- (d) A woman takes out a home improvement loan for 11,000 over 5 years. She makes monthly repayments in arrears and the bank charges an effective rate of interest of 6% pa.
- i. What is the monthly repayment? [2 marks]
  - ii. How much interest does she pay in the third year? [2 marks]
  - iii. How much capital is repaid in the 20th instalment? [2 marks]
  - iv. At the end of the fourth year she decides to make further improvements to her house and wants to borrow another 4,000 at that stage. If her total balance is to be repaid over 3 years by level monthly payments and there is no alteration to the interest rate, how much is each payment? [3 marks]

## QUESTION FIVE

- (a) A businessman is considering a certain project, which involves setting up a shop. He estimates that the venture will require an initial outlay of ksh. 200000 and a further outlay of ksh. 1000000 after one year. There will be an estimated inflow of ksh. 300000 per annum payable continuously for ten years beginning in three years' time and a final inflow of ksh. 600000 when the project ends in thirteen years' time. Measuring time in years,
- describe the net cashflows associated with this venture and illustrate the position on a diagram. [3 marks]
  - find the net present value,  $NPV(i)$  for this business venture. [5 marks]
- (b) Suppose that an account offers a nominal interest rate of 8% p.a. payable quarterly.
- What is the AER? [2 marks]
  - What if the nominal rate is the same, but interest is payable monthly? Weekly? Daily? [6 marks]
- (c) A continuous cashflow is described by the rate  $\rho(t) = a + be^{\alpha t}$  per annum, where  $a$ ,  $b$  and  $\alpha$  are constants, and time is measured in years, starting at  $t = 0$ . Let  $c_1$ ,  $c_2$  and  $c_3$  be the amounts paid during the 1st, 2nd and 3rd year, respectively. It is known that

$$\frac{c_3 - c_2}{c_2 - c_1} = 1.05$$

Show that  $\alpha = \ln 1.05$  regardless of  $a$  and  $b$ . [4 marks]