



JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY
SCHOOL OF MATHEMATICAL & ACTUARIAL SCIENCE
UNIVERSITY EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE
(ACTUARIAL) WITH IT
1ST YEAR 1ST SEMESTER 2013/2014 ACADEMIC YEAR
CENTRE: MAIN

COURSE CODE: SAC 103

COURSE TITLE: MATHEMATICS MODELLING

EXAM VENUE: LR 1

STREAM: (BSc. Actuarial)

DATE: 14/4/2014

EXAM SESSION: 2.00 – 4.00 PM

TIME: 2 HOURS

Instructions:

- 1. Answer ALL questions in section A and ANY other 2 questions in section B**
- 2. Candidates are advised not to write on the question paper.**
- 3. Candidates must hand in their answer booklets to the invigilator while in the examination room.**

SECTION ONE (COMPULSORY 30Marks)

1. You have a balance of \$5,000 on a credit card which charges 2% interest per month. You promise to pay p dollars a month to the credit card company and not make any new charges.
 - (a) Formulate a model in terms of p which allows you to pay off the credit card in 10 years.
(3 marks)
 - (b) Solve your model analytically to find the value of p (to the nearest cent) which will allow you to pay off the credit card in exactly 10 years.
(5 marks)
 - (c) True or False: If p is less than \$100, you will never be able to pay off your credit card. Justify your answer.
(5 marks)
2. Consider the following predator-prey model for two species x and y . Assume that when the predator and the prey meet, it is good for the population of the predator and bad for the population of the prey.
$$\begin{aligned}x_{n+1} &= 0.8x_n + 0.001x_ny_n \\ y_{n+1} &= y_n + 0.0001y_n(3000 - y_n) - 0.004x_ny_n\end{aligned}$$
 - (a) Identify which variable represents the predator and which represents the prey. Explain your answer.
(3 marks)
 - (b) According to the model, what happens to the population of the predator if there are no prey present?
(5 marks)
 - (c) According to the model, will the population of the prey experience unlimited population growth if there are no predators present?
(4 marks)

SECTION TWO

- 1.(a) Your parents are considering a 30-year, \$100000 mortgage that charges 0.5% interest each month. They want to pay \$ p monthly so that the mortgage (loan) is paid off after 360 payments (i.e., 30 years).
 - (1) Set up and solve the model with the dynamical system (i.e., difference equation).
 - (2) Find the monthly payment \$ p .
(5 marks)

- (b) It is known that, for certain income brackets, money deposited in an individual retirement Account (IRA) and its interest are tax-deferred. Karen Smith sets up an IRA at the beginning of the year and deposited \$3,000. Karen will deposit \$2,000 at the end of each year, and the interest rate is 6% compounded annually.
- Model this situation by a difference equation.
 - What will be the future value of the account at Karen's retirement age of 60 if she starts the IRA at the age of 25? (5 marks)

(c) Solve the following differential equation.

$$\frac{dy}{dx} = 6y^2x \quad y(1) = \frac{1}{25} \quad (10 \text{ marks})$$

2. (a) Assume you open an account that pays 4% compounded annually and deposit \$2,000. You will deposit 2% more into your account than you deposited in the previous year.
- Model this situation by a difference equation.
 - What is the total amount in the account after 20 years? (5 marks)

- (b) You want to buy a new car that costs \$16,000. The dealer offers \$4,000 for a trade-in with a down payment of \$2,000. You borrow \$10,000 from a bank for 5 years at an annual rate of interest 12% compounded monthly. The payments are made at the end of each month. What will the monthly payment be? (5 marks)

c) Suppose it is known that the cells of a given bacterial culture divide every 3.5hrs on average. If there are 500 cells in a dish to begin with, how many will there be after 12 hours? (10 marks)

3. a) A body was discovered at 6.00 a.m one morning in an apartment. The temperature of the body was 32°C . The room was air conditioned at a constant 20°C . Assuming the difference between the body and the surrounding temperature dropped at 6% per hour, what time did the murder occur? (Take normal body temperature to be 38°C) (10 marks)

- b) The population of a community is known to increase at a rate proportional to the number of people present at a time t . If the population has doubled in 6 years, how long will it triple? (10 marks)

4. a) A steak of meat is removed from a freezer and put into the refrigerator to thaw. The freezer is kept at -10°C and the fridge is kept at 4°C . After 4 hours, the temperature of the steak was -6°C . When will the steak be thawed to 2°C ? (10 marks)

- b) A certain investment grows from a balance of \$2500 to \$4132 in 12 years.

The investment account offers continuously compounded interest. What is the annual interest rate? (5 marks)

c) A 50 kg mass is shot from a cannon straight up with an initial velocity of 10m/s off a bridge that is 100 meters above the ground. If air resistance is given by $5v$ determine the velocity of the mass when it hits the ground. (5 marks)