JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

UNIVERSITY EXAMINATION 2012/2013

1ST YEAR 1ST SEMESTER EXAMINATION FOR THE DEGREE OF BSC. (COMPUTER SECURITY & FORENSIC)

(REGULAR)

COURSE CODE: SMA 3113

TITLE: LOGICAL FUNCTIONS

DATE: 30/4/2013 **TIME: 9.00-11.00AM**

DURATION: 2 HOURS

INSTRUCTIONS

1. **This paper contains SIX (6) questions**
2. **Answer question 1 (Compulsory) and ANY other 2 Questions**
3. **Write all answers in the booklet provided**

QUESTION ONE (30 marks)

a) Define the terms: Contrapositive, Logical equivalence and Tautology **[3 marks]**

b) Let p, q and r be the propositions:

p : You have malaria, q : You miss the CAT, r : You pass the course.

Express each of the following propositions as an English sentence.

(i) $(p \to \neg r) \vee (q \to \neg r)$ (ii) $(p \wedge q) \vee (\neg q \wedge r)$ **[4 marks]**

c) Find bitwise OR, bitwise AND, and bitwise XOR of the following pair of bit strings: 11 0010110010 and01 0001110101 [3 marks]

d) Construct a truth table for the compound proposition: $(p \oplus q) \vee (p \oplus \neg q)$ **[4 marks**]

e) Use mathematical induction to prove that

$1 + 2 + 2^2 + 2^3 + \cdots + 2^n = 2^{n+1} - 1$ **[5 marks]**

f) How many bit strings are there of length nine? **[2 marks**]

g) Define the term "Cryptography" and hence , use Caesars' encryption to encrypt the message **MEET YOU IN THE PARK** **[4 marks]**

h) Let the universal set be: $U = \{1,2,3,4,5,6,7,8,9,10\}$

Express the following strings with bit strings:

(i) $A = \{2,3,4,7,8,9\}$ (ii) $B = \{1,3,6,10\}$ (iii) $\overline{A} \cap \overline{B}$ (iv) $D = A - B$ **[5 marks]**

QUESTION TWO (20 marks)

a) Find the truth values of (i) $\forall x \exists y \neg P(x, y)$ (ii) $\neg \exists x \forall y P(x, y)$ **[4 marks]**

b) Each user of a computer system has a password which is six to eight characters long, where each character is an upper case letter or a digit. Each password must contain at least one digit. How many possible passwords are there? **[10 marks**]

c) What is the power set P(S) of S = {F,G,H,K} **[6 marks]**

QUESTION THREE (20 marks)

a) Show that $(p \wedge q) \vee (p \wedge r)$ and $p \wedge (p \vee r)$ are logically equivalent. **[7 marks]**

b) Prove that if A and B are nonempty sets, then $\overline{A \cup B} = \overline{A} \cap \overline{B}$ **[8 marks**]

c) Evaluate the quality: -97 **mod** 11 **[5 marks]**

QUESTION FOUR (20 marks)

a) What is the value of k after the following code has been executed?

```
k :  =  0
for     i1 :  =  1   to   n1
        k :   =  k   +    1
for     i2 :     1   to   n2
        k      =  k   +    1
        .
        .
        .
for     im :   =  1   to   nm
        k      =  k   +    1
```

[5 marks]

b) How many different car number plates are available if each number plate contains a sequence of three letters followed by the three digits. The first letter must be a k and no sequence of letters are prohibited? **[5 marks]**

c) Use mathematical induction to prove that $n^3 - n$ is divisible by 3 whenever n is a positive integer. **[5marks]**

d) What is the least number of area codes needed to guarantee that 40 million phones in a country have distinct 10-digit telephone numbers? Assume that telephone numbers are of the form $NXX - NXX - XXXX$, where the first three digits form the area code, N represents a digit from 2 to 9 inclusive, and X represents any digit. [**5 marks]**

QUESTION FIVE (20 marks)

a) Let $P(x): x$ is a professor, $Q(x): x$ is ignorant, $R(x): x$ is careless. Express each of the following statements using quantifiers, logical connectives, and $P(x)$, $Q(x)$, $R(x)$ where the universe of discourse is the set of all people.

(i) No professors are ignorant (ii) All ignorant people are careless (iii) No professors are careless (iv) Does (iii) follow from (i) and (ii)? If not, is there a correct inclusion? **[9 marks]**

b) Decrypt the message: **HDWGLPVXP** using Caesar's cipher **[6 marks]**

c) Define the 'floor' and 'ceiling' functions and hence find the values of (i) $\left\lfloor -7/8 \right\rfloor$ and (ii) $\lceil -1 \rceil$ [5 marks]