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BACHELOR OF SCIENCE -ACTUARIAL SCIENCE

YEAR ONE SEMESTER ONE

SAS 103: INTRODUCTION TO PROBABILITY THEORY

Instructions: Answer question 1 and any other 2 questions

QUESTION 1(30mks)

a) Each team in a basketball league plays 20 games in one tournament. Event A is the event that Team 1 wins 15 or more games in the tournament. Event B is the event that Team 1 wins less than 10 games. Event C is the event that Team 1 wins between 8 to 16 games. Of course, Team 1 can win at most 20 games. Using words, what do the following events represent? (4mks)

(a) $A \cup B$ and $A \cap B$.
(b) $A \cup C$ and $A \cap C$.
(c) $B \cup C$ and $B \cap C$.
(d) A^c, B^c, and C^c.

b) Consider the sample space S = {1, 2, 3}. Suppose that P({1, 2}) = 0.5 and P({2, 3}) = 0.7. Is P a valid probability measure? Justify your answer. (3mks)

c) Suppose there's 40% chance of colder weather, 10% chance of rain and colder weather, 80% chance of rain or colder weather. Find the chance of rain. (2mks)

d) Suppose that out of 500 computer chips there are 9 defective. Construct the probability tree of the experiment of sampling two of them without replacement. (4mks)

e) In a state assembly, 35% of the legislators are Democrats, and the other 65% are Republicans. 70% of the Democrats favor raising sales tax, while only 40% of the Republicans favor the increase. If a legislator is selected at random from this group, what is the probability that he or she favors raising sales tax? (4mks)

f) Let A denote the event "student is female" and let B denote the event "student is French". In a class of 100 students suppose 60 are French, and suppose that 10 of the French students are females. Find the probability that if I pick a French student, it will be a female, that is, find P(A|B). (3mks)

g) Suppose an individual applying to a college determines that he has an 80% chance of being accepted, and he knows that dormitory housing will only be provided for 60% of all of the accepted students. What is the probability that a student will be accepted and will receive dormitory housing?

(2mks)

h) A company that manufactures video cameras produces a basic model (B) and a deluxe model (D). Over the past year, 40% of the cameras sold have been of the basic model. Of those buying the basic model, 30% purchased an extended warranty, whereas 50% of all deluxe purchases do so. If you learn that a randomly selected purchaser has an extended warranty, how likely is it that he or she has a basic model?

(5mks)

i) An actuary studied the likelihood that different types of drivers would be involved in at least one collision during any one-year period. The results of the study are presented below.

Type of driver	Percentage of all drivers	Probability of at least one collision
Teen	8%	0.15
Young adult	16%	0.08
Midlife	45%	0.04
Senior	31%	0.05
Total	100%	

Given that a driver has been involved in at least one collision in the past year, what is the probability that the driver is a young adult driver? (3mks)

QUESTION 2 (20mks)

a) Let X be a random variable with probability distribution table given below

x	0	10	20	50	100
P(X=x)	0.4	0.3	0.15	0.1	0.05

Find P(X < 50). (4mks)

b) Let X be a discrete random variable with the following probability table

x	1	5	10	50	100
P(X=x)	0.02	0.41	0.21	0.08	0.28

Compute

$$P(X > 4 | X \leq 50).$$

(4mks)

c) A committee of 4 is to be selected from a group consisting of 5 men and 5 women. Let X be the random variable that represents the number of women in the committee. Create the probability mass distribution.
(4mks)

d) Suppose that an insurance company has broken down yearly automobile claims for drivers from age 16 through 21 as shown in the following table. (4mks)

Amount of claim	Probability
$ 0	0.80
$ 2000	0.10
$ 4000	0.05
$ 6000	0.03
$ 8000	0.01
$ 10000	0.01

How much should the company charge as its average premium in order to break even on costs for claims?

e)

Let X be a random variable with $E(X) = 6$ and $E(X^2) = 45$, and let $Y = 20 - 2X$. Find $E(Y)$ and $E(Y^2) - [E(Y)]^2$.

(4mks)

A life insurance salesman sells on the average 3 life insurance policies per week. Use Poisson distribution to calculate the probability that in a given week he will sell

(a) Some policies

(b) 2 or more policies but less than 5 policies.

(c) Assuming that there are 5 working days per week, what is the probability that in a given day he will sell one policy? (4mks)

QUESTION 3 (20mks)

i) From past experience it is known that 3% of accounts in a large accounting population are in error.
(a) What is the probability that 5 accounts are audited before an account in error is found?
(b) What is the probability that the first account in error occurs in the first five accounts audited?
(5mks)

ii) Suppose we are at a rifle range with an old gun that misfires 5 out of 6 times. Define "success" as the event the gun fires and let X be the number of failures before the third success. Then X is a negative binomial random variable with parameters (3, 1/6). Find E(X) and Var (X).
(5mks)

iii) The probability mass function of X, the weekly number of accidents at a certain intersection is given by p(0) = 0.40, p(1) = 0.30, p(2) = 0.20, and p(3) = 0.10.
(a) Find the CDF of X.
(b) Find the probability that there will be at least two accidents in any one week. (5mks)

iv) Let X be the time (in hours) required to repair a computer system. We assume that X has an exponential distribution with parameter (5mks)
$\lambda = \frac{1}{4}$.

Find
(a) the cumulative distribution function of X.
(b) P(X > 4).
(c) P(X > 10|X > 8).

QUESTION 4 (20mks)

i) The distance between major cracks in a highway follows an exponential distribution with a mean of 5 miles.
(a) What is the probability that there are no major cracks in a 20-mile stretch of the highway?
(b)What is the probability that the first major crack occurs between 15 and 20 miles of the start of inspection?
(c) Given that there are no cracks in the first 5 miles inspected, what is the probability that there are no major cracks in the next 15 miles? (5kms)

ii) A store has 80 modems in its inventory, 30 coming from source A and the remainder from source B. Of the modems from source A, 20% are defective. Of the modems from source B, 8% are defective.

Calculate the probability that exactly two out of a random sample of five modems from the store's inventory are defective. (4mks)

iii) Based on past experience, a company knows that an experienced machine operator will produce a defective item 1% of the time. Operators with some experience have a 2:5% defect rate, and new operators have a 6% defect rate. At any one time, the company has 60% experienced operators, 30% with some experience, and 10% new operators. Find the probability that a particular defective item was produced by (10mks)

a) a new operator.
b) An operator with some experience
c) An experienced operator

QUESTION 5 (20MKS)

a) An insurance company issues 1250 vision care insurance policies. The number of claims filed by a policy holder under a vision care insurance policy during one year is a poisson random variable with mean 2. Assume the numbers of claims filed by distinct policy holders are independent of one another.

(10mks)

What is the approximate probability that there is a total of between 2450 and 2600 claims during a one year period?

b) An insurance company sells two types of auto insurance policies: Basic and Deluxe. The time until the next Basic policy claim is an exponential random variable with mean two days. The time until the next Deluxe policy claim is an independent exponential random variable with mean three days. What is the probability that the next claim will be a Deluxe policy claim? (10mks)