CHUKA UNIVERSITY

COLLEGE

UNIVERSITY EXAMINATIONS

THIRD YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF EDUCATION (ARTS),

MATH 344: ESTIMATION THEORY

STREAM: B.ED (ARTS) Y3S2 **TIME: 2 HOURS**

DAY/DATE: TUESDAY 19/4/2011 **2.30P.M. - 4.30 P.M.**

INSTRUCTIONS:

Attempt Question One and any other two.

QUESTION ONE (30 MARKS)

(a) Given that $S^2 = \frac{\sum (yi - \bar{y})^2}{n}$, show that the sample variance S^2 is unbiased estimator of the population variance δ^2 as $n \rightarrow \infty$. [5 marks]

(b) Let x_1, x_2, x_3, x_4 be four independent sample observation of a Poisson distribution with parameter λ. Show that T = $\frac{x_1 + 4x_2 + 5x_3 2x_4}{12}$ is an unbiased estimator of λ. [4 marks]

(c) Given that the p.d.f of a random variable X is defined by

$$f(x,\theta = \begin{cases} \frac{1}{\theta} & ,0 \le x \le \theta \\ 0 & ,otherwise \end{cases}$$

Find the constant C such that C$\bar{x}$ is unbiased estimator of Cθ. [4 marks]

(d) Find the sufficient statistic for δ^2 given that the random variable X is normally distributed with X~N(μ, ∂^2) and μ is known. [5 marks]
(Hint: Use the Pitman – Koopman from of distribution).

(e) (i) State two conditions that must be satisfied for an estimator T to be considered consistent. [2 marks]

(ii) Given that X is a random variable from a Bernoulli distribution, show that x/n is a consistent estimator of P. [3 marks]

(f) Given that Y~N(μ, ∂^2) with ∂ being known. Find I (μ). [4 marks]

(g) Let Y_1, Y_2, Y_3 be independent sample values from a Poisson population with parameter θ. If T1 = $\frac{2Y_1 + 3Y_2 + 4Y_3}{9}$ and T_2 = $\frac{Y_1 + Y_2 + Y_3}{3}$ show that T_1 and T_2 are unbiased estimator of θ and also find the efficiency of T_1 in relation to T_2. [3 marks]

QUESTION TWO (20 MARKS)

(a) Let X~N(μ, ∂^2) with μ being known, show that S^2 is the minimum variance bound unbiased estimator of ∂^2. [10 marks]

(b) (i) State five properties of maximum likelihood estimation. [5 marks]

(ii) Given the p.d.f

$$f(x, \mu, \partial) = \frac{1}{\partial\sqrt{2\pi}} \exp\left\{-\tfrac{1}{2}\frac{x-\mu)^2}{\partial^2}\right\}_{\substack{-\infty<x,<\infty \\ \partial>0}}$$

Find the M.L.E of ∂^2. [5 marks]

QUESTION THREE (20 MARKS)

(a) Describe briefly the method of moments for estimation of parameter θ in the p.d.f

$f(x, \theta) = \theta x^{\theta-1}$, $0 \leq x \leq 1$, $0>0$

using the method of moments. [5 marks]

(b) Consider a random variable X with a Cauchy distribution defined by the p.d.f

$$f(x,\theta) = \frac{1}{\overline{x}} \cdot \frac{1}{1+(x-\theta)^2} \quad -\infty < x < \infty$$

Find the information function I (θ) . [10 marks]

QUESTION FOUR (30 MARKS)

(a) Let X_2, X_2 be a random sample of size Z from the distribution

$$f(x,\theta) = \frac{1}{\theta} e^{-x/\theta}, \quad 0 \le x < \infty, \theta > 0$$

Show that T = $\frac{4}{\pi}\sqrt{x_1 x_2}$ is unbiased estimator of θ. [5 marks]

(b) Let $\overline{X}$ and $\overline{Y}$ denote the means of two independent random sample each of size n from a normal population with parameters (μ, ∂^2) and (μ_2, ∂^2). Find the size of the sample such that

$$\text{P}\left[(\overline{X} - \overline{Y})\right]\partial/5 < \mu_1 - \mu_2 < (\overline{X} - \overline{Y}) + \partial/5 = 0.9$$ [5 marks]

(c) Let X be a Poisson variate with parameter θ. Verify whether T = $\sum xi$ is a sufficient statistic of θ by finding the conditional distribution of $f(x_1, x_2 \cdots x_n \mid T = t) = \frac{P(x_1, x_2, \cdots, x_n)}{P(T = t)}$ [10 marks]
