BONDO UNIVERSITY COLLEGE

3^{rd} ***YEAR*** 1^{st} ***SEMESTER EXAMINATION***[SCHOOL BASED]

SMA 300: REAL ANALYSIS I

INSTRUCTION: Attempt question one (**COMPULSORY**) and any other TWO questions only.

QUESTION ONE(*COMPULSORY*) [30 MARKS]

(a). Define the following terms: Monotonic sequence, Finite set and Cardinal number. (5 marks)

(b). Define equivalent sets and show that the set of natural numbers is equivalent to the set of even natural numbers. (5 marks)

(c). Distinguish between absolute convergence and conditional convergence. (4 marks)

(d). State Minkowski's inequality for an n-dimensional real space. (3marks)

(e). If $L = \{(x, y) \in \mathbb{R}^n : 7x + 3y = 14\}$ and $r = (2, -1)$, find $d(r, L)$. (4 marks)

(f). State the Comparison Test and use it to test the convergence of $f_n = \frac{1}{n^2+k^2}$. (5 marks)

(g). Given the set $V = \{v : v \; is \; a \; digit\}$, find the cardinality of V and the subset of V containing nonzero prime numbers. (4 marks)

QUESTION TWO [20 MARKS]

(a). Explain the countability of a set. (2 marks)

(b). Is every subset of a countable set countable? Prove. (8 marks)

(c). Prove that the union of a countable number of sets is countable. (10 marks)

QUESTION THREE [20 MARKS]

(a). Define the term metric. (3 marks)

(b). If $X = \{(x, y) \in \mathbb{R}^2 : xy = 1\}$ and $Y = \{(x, y \in \mathbb{R}^2 : y = 0)\}$, find $d(X, Y)$. (4 marks)

(c). Describe four features of a step function. (4 marks)

(d). Let $\mathbb{R}$ be the set of of real numbers. Is $d : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ defined by $d(x, y) = |x - y|$, for all $x, y \in \mathbb{R}$ a metric on $\mathbb{R}$? Show your working.
(9 marks)

QUESTION FOUR [20 MARKS]

(a). Define convergence of a sequence. (2 marks)

(b). Show that the constant sequence is convergent. (7 marks)

(c). Prove the uniqueness of the limit of a function. (11 marks)

QUESTION FIVE [20 MARKS]

(a). Define topology. (3 marks)

(b). Let $P = \{1, 2, 3\}$, $\hbar = \{\emptyset, \{1\}, \{1, 2\}, \{1, 3\}, P\}$ and $\hbar' = \{\emptyset, \{1\}, \{2\}, \{1, 2\}, \{2, 3\}, P\}$.

(i). Determine whether $\hbar$ is a topology on P. (8 marks)

(ii). Determine whether $\hbar \cup \hbar'$ is a topology on P. (9 marks)