

JOUST

KOSELE

SMA200: CALCULUS II.

QUESTION 1 [30MKS]:[COMPULSORY]:

1. (a) Evaluate the integral (3mks)

$$\int \tan^{-1} x \, dx$$

(b) Evaluate the following integrals:

(i) $\int \frac{dx}{4x+5}$ (3mks).

(ii) $\int \frac{1}{x^2-2x+5} \, dx$ (4mks)

(c) Find the area between the curves $f(y) = (3 - x^2)$ and $g(y) = (y+1)$. (4mks).

(d) Find the intercepts of $f(x) = x^2 - 3x + 2$ and show that $f'(x) = 0$ at some point between the two curves. (5mks).

(e) Find the volume of the solid formed by revolving the region bounded by $f(x) = (2 - x^2)$ and $g(x) = 1$ about the line $y = 1$. (5mks).

(f) Find the arc length of the curve given by $f(x) = \frac{2}{3}x^{1.5} + \frac{1}{x}$ (6mks).

QUESTION 2.[20MKS]:

(a). Find the area of the surface obtained by revolving the curve $f(x)=x^3$ on the interval $0 \leq x \leq 1$ about the x-axis. (7mks).

(b) Evaluate the integral:

$$\int x^2 e^x dx \text{ (6mks).}$$

(c) Express as the following fractions as a sum of partial fractions :

(i) $\frac{x-1}{3x^2-11x+10}$.

(ii) $\int \frac{x-1}{3x^2-11x+10} dx$ (7mks).

QUESTION 3[20MKS]:

(a) Evaluate the $\lim_{x \rightarrow 0} \frac{e^{2x}-1}{x}$ (3mks).

(b) Using the change of variable $t = \tan x$; Find $\int \frac{1}{1 + \sin^2 x} dx$ (6mks).

(c) Taking $I_n = \int_0^{\frac{\pi}{2}} \cos^n x dx$; show that $\int_0^{\frac{\pi}{2}} \cos^2 x dx = \left(\frac{n-1}{n}\right) I_{n-2}$, (11mks).

Where $n \geq 2$

QUESTION 4. (20MKS):

(a).(i) Prove that $\int_0^1 x^2 e^{2x} = \frac{e^2}{4} - \frac{1}{4}$ (4mks).

(ii) $\int \frac{2x+3}{x^2+2x+3} dx$ (6mks).

(b) $\int_1^2 \frac{x^2+2}{x+1} dx$ (6mks).

(c) If $f(x) = (5 - \frac{4}{x})$, find all c in the interval $(1, 4)$ such that

$$f'(c) = \frac{f(4) - f(1)}{(4-)} (4\text{mks}).$$

QUESTION 5 [20MKS]:

(a) Evaluate the following:

(i) $\int \sin^5 x \, dx$ (5mks)

(ii) Find the approximate value of $\sqrt[10]{1.01}$ (5mks)

(b) (i) Using the first four terms, show that the approximate value of

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots (5\text{mks})$$

(ii) Calculate the volume of the solid formed by revolving the region

Bounded by the curve $y = \sqrt{x}$ and $y = x^2$ (6mks).

