JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

UNIVERSITY EXAMINATIONS 2013

3RD YEAR 1ST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION ARTS WITH IT

(S BASED KOSELE)

COURSE CODE:SMA 300

TITLE: REAL ANALYSIS

DATE: **TIME:**

DURATION: 2HOURS

INSTRUCTIONS

1.This paper contains FIVE (5) questions.

2.Answer question 1 (Compulsory) and ANY other 2 questions.

3.Write all answers in the booklet provided.

QUESTION 1

(a) Let (X,d) be a metric space and let A,B$\subseteq$ X. Explain what is meant by each of the

following statements:

(i) p is an interior point of A. **(2mks)**

(ii) A is an open subset of X. **(2mks)**

(iii) B isnot open in X. **(2mks)**

(b) State any differences between collections of open and closed subsets with respect to the operations of union and intersection. **(2mks)**

(c) Show that the open interval (0,1) is not compact. **(5mks)**

(d) Prove that a real valued continous function on a closed interval attains its minimum and maximum values on the interval . Use an example to show that it fails when the interval is not closed. **(10mks)**

(e) Let (X,d) be a sequence defined by $x_n = \sqrt[n]{np}$ for all n $\in$ J^+ where p > 0 is fixed. Show that x_n converges to 1. **(4mks)**

(f) Let (X,d) be a metric space and (x_n) be a sequence in X. If (x_n) is convergent then (x_n) is cauchy, prove. **(3mks)**

QUESTION 2

(a) State the axioms of a metric defineda non empty set X. Verify these axioms for the standard metric on **R**. **(15mks)**

(b) Let (X,d) be a metric space. Show that $|d(x,y)-d(x,z)| \leq d(y,z)$ for all x,y,z $\in$ X.**(5mks)**

QUESTION 3

(a) Let (X,d)be a metric space and A$\subseteq$X. Give the definition of the interior of A denoted by A^0. Hence prove that A^0 is the largest open set contained in A. **(6mks)**

(b) Let (X,d) be a metric space and $x_0 \in X$ be a fixed point.Then the neighbourhood

$N(x_0,r)$ or the ball $B(x_0,r)$ for some r>0 is open, prove. **(6mks)**

(c) Given that X=**R** and A={x$\in$**R**:0<x<1}. Show that 1/4 is an interior point. **(2mks)**

(d) Prove that the closure of a set A denoted by $\bar{A}$ is a closed set. **(6mks)**

QUESTION 4

(a) Given the function f:[o,1]→**R**defined by f(x)= x^2. Show that f is uniformly continous. **(6mks)**

(b) Give the definition of continuity of a function defined on a metric space in terms of neighbourhoods. **(3mks)**

(c) Give the definition of uniform continuity of a function defined on a metric space. Also give an example of a uniformly continous function and state the condition under which pointwise continuity implies uniform continuity. **(9mks)**

QUESTION 5

(a) Let $(F_\alpha)_{\alpha \in I}$ be a family of closed subsets ina metric space (X,d). Then the intersection of the F_α is also closed, Prove. **(10mks)**

(b) Let (X,d) be a metric space and $(F_i)^n_{i=1}$ be a finite family of closed subsets of X. Prove that $U^n_{i=1}$ is also closed. **(10mks)**