COURSE TITLE: **ODE I**

COURSE CODE: **SMA 301**; TIME: 2 HOURS

KOSELE LC; AUGUST 2013

Answer question ONE and any other TWO questions.

QUESTION 1

(a) Classify the following differential equations.

(i) $y' = y$

(ii) $y'' = 4y$

(iii) $(y') - 3y = e^x$

[3mks]

(b) Verify that $y = cx^2$ is the solution of $xy' - 3y = 0$ hence find the particular solution given that $y(3) = 2$ **[6mks]**

(c) Show that $f(x, y) = x^2y - 4x^3 + 3xy^2$ is homogenous function of degree three **[4mks]**

(d) Solve the differential equation $y'' - 4y = 0$ **[5mks]**

(e) Test for exactness in $(2xy - 3x^2)dx + (x^2 - 2y)dy = 0$. **[6mks]**

(f) Find the general solution of $\frac{dy}{dx} + 5 = 6x$, $y(1) = 2$ **[6mks]**

QUESTION 2

(a) Show that $(y^2 + ycosx)dx + (3xy + 2sinx)dy = 0$ is not exact then find an integrating factor and solve it. **[10mks]**

(b) Find the equation of a curve that passes through the point $(1, 3)$ and has a slope of $\frac{y}{x^2}$ at the point (x, y) **[7mks]**

(c) Show that $y_1(x) = x^{1/2}$ and $y_2 = x^{-1}$ form a fundamental set of the solution of $2x^2y'' + 3xy' - y = 0$, $x > 0$ **[3mks]**

QUESTION 3

(a) Find the general solution of the equation

$y'' - 2y' - 8y = xe^x + 4$ **[10mks]**

(b) Show that the following differential equation is homogenous and solve it.

$(2x + y)dx + (x - 2y)dy = 0$ **[10mks]**

QUESTION 4

(a) Check for exactness and solve the equation.

$(5x^2+3x^2y^3-2xy^3)dx+(2x^3y-3x^2y^2-5y^4)dy = 0$ **[15mks]**

(b) Newton's law of cooling states that if an object is hotter than the ambient temperature, then the rate of cooling of the object is proportional to the temperature difference

$$\frac{d\Theta}{dt} = -k(\Theta - A)$$

with $\Theta(t_0) = \Theta_0$, where $\Theta(t)$ is the object's temperature, A is the ambient temperature (a constant)and k is a positive constant. Show that by using the initial condition and rearranging, this is a first-order linear ODE results in,

$$\Theta(t) = A + (\Theta_0 - A)e^{-k(t-t_0)}$$

[5mks]

QUESTION 5

(a) Use the method of variation of parameters to solve

$y'' - 2y' + y = \frac{e^x}{2x}$ for $x > 0$ **[10mks]**

(b) Given the logistic equation:

$$\frac{dp}{dt} = \mu p\left(1 - \frac{p}{p_\infty}\right)$$

with $p(t_0) = p_0$ solve the separable ODE and show that,

$$p(t) = \frac{p_\infty}{1 + (\frac{p_\infty}{p_0} - 1)e^{-\mu(t-t_0)}}$$

[10mks]