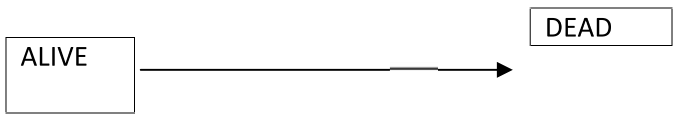
SAC 203: FUNDAMENTALS OF ACTUARIAL MATHEMATICS II

Instructions: Answer Question One and other Two

Question one: compulsory **[30marks]**

(a)Mortality over a short age range may be modeled by the following two state models



The force of transition from alive to death is μ and is constant. You observe the following statistics in respect of N independent lives
d = the number of deaths (which is the realization of the variable D)
v= the total time under observation for all N lives (which is the realization of the variable D)
Required
- (i) The likelihood function for μ
- (ii) The variance of $\hat{\mu}$, the MLE of μ as asymptotically distribute **[17Marks]**

(b) An insurance company is investigating the mortality of its annuity policyholders. It's proposed that the crude mortality rates be graduated for use in future premiums calculation
(i) Suggest with reasons, a suitable method of graduation in this case and describe how you would graduate the crude rates
(ii) Comment on any further considerations that the company should take into account before using the graduated rates for premium calculation **[13 marks]**

Question Two
(i)Estimate E_{55}^{c} based on the following table

Calendar year	Population aged 55last birthday on 1st January
2001	46,233
2002	42,399
2003	42,618
2004	42,020

(ii). the disreputable insurance company, Honest Sid's Mutual had mixed fortunes in the year in the year 2003. At both the start and the end of the year 547 policies were in force in respect of policyholders aged 40 but these figures do not tell the whole story. There was a diverse publicity early in the year linking the company's investment managers with a gambling syndicate. As a result many policyholders took their money elsewhere. Following a successful marketing campaign offering a free toaster to all applicants, the number of policyholders aged 40 rose from 325 at 1st June 2003 to 613 at 1st 2003. Calculate E_{40}^{c} approximately **[20 Marks]**

Question Three

(i) Describe the difference between central exposed to risk and the initial exposed to risk.

The following data come from an investigation of the mortality of participants in a dangerous sport during the calendar year 2005

Age x	Number of lives aged x last birthday on		Number of deaths during 2005 to persons aged x last birthday
	1/Jan/2005	1/Jan/2006	
22	150	160	20
23	160	155	25

(ii) (a) Estimate the initial exposed to risk at ages 22 and 23

(b) Hence estimate q_{22} and q_{23}

Suppose that in this investigation, instead of aggregate data we had individual level data on each person's date of death, date of birth and date of exit from the observation(if exit was for reasons other than death

(iii) Explain how you would calculate the initial exposed to risk for lives aged 22 years last birthday

[20 marks]

Question Four

An investigation took place into the mortality of pensioners. The investigation began on 1st Jan 2003 and ended on 1st Jan 2004. The table below gives the data collected in this investigation for 8 lives

Data of Birth	Date of entry into observation	Date of exit from observation	Whether or not exit was due to death(1) or other reason(0)
1april 1932	1jan 2003	1jan 2004	0
1october 1932	1jan2003	1jan2004	0
1November 1932	1march2003	1september2003	1
1january 1933	1march 2003	1june2003	1
1january 1933	1june2003	1september2003	0
1 March1933	1september2003	1january2004	0
1June 1933	1january 2003	1january2004	0
1october 1933	1june2003	1january2004	0

The force of mortality, u_{70} , between exact ages 70 and 71 is assumed to be constant.

(i) Estimate the constant force of mortality, 70, using a two-state model and the data for the 8 lives in the table, hence or otherwise estimate q_{70}

(ii) Show that the maximum likelihood estimate of the constant force, u_{70}, using a Poisson model of mortality is the same as the estimate using the two-state model.

(iii) Outline the differences between the two-state model and the Poisson model when used to estimate transition rates. **[20Marks]**

Question Five

(i) State the assumptions underlying the binomial Mortality Model **[3]**

(ii) A cat has nine lives so the cat will not die until it has lost all nine of its lives. The probability that a Cat looses a life is 20% per week. Assuming that the mortality of each life follows the binomial Model. Calculate the following

(a) The probability that a cat who currently lost none of its lives will die during the next 10weeks

(b) The probability that this cat during the fifth week **[7]**

(iii) Prove that the probability that exactly x decrements will occur in a population consisting initially of n individuals subject to single decrement with rate q per annum is

$$\binom{n}{x} q^x (1-q)^{n-x}$$

Hence or otherwise

(a) Prove that the maximum likelihood estimate of q for the binomial Model equals the number of decrements divided by the initial population

(b) The maximum likelihood estimate of the parameter μ for the Poisson Model is an unbiased estimator of the force of Decrement **[10]**

[20Marks]