JARAMOGI OGINGA ODINGA UNIVERSITY

OF SCIENCE & TECHNOLOGY

UNIVERSITY EXAMINATIONS 2012/2013

2ND YEAR 1ST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE ACTUARIAL SCIENCE

(REGULAR)

COURSE CODE: SAS 303

COURSE TITLE: ESTIMATION THEORY

DATE: 12/8/2013 **TIME: 9.00-11.00 AM**

DURATION: 2 HOURS

INSTRUCTIONS

1. **This paper consists of 5 Questions.**
2. **Answer Question 1 (Compulsory) and any other 2 questions.**
3. **Write your answers on the answer booklet provided.**

QUESTION ONE (COMPULSORY) **[30 Marks]**

(a)(i)The random variable T_{xy} represents the time to failure of the joint life status(xy), x is subject to constant force of mortality of 0.02 and y is subject to a constant force of mortality 0.03. (x)And (y) are independent with respect to mortality.

Calculate the value of $E(T_{xy})$

(ii)A population is subject to two modes of decrement α andβ where $q_x^{\beta} = {}^1/_3 + {}^1/_4\, q_x^{\alpha}$ Derive from first principles$(aq)_x^{\beta}$. State clearly the assumptions you havemade **[13 Marks]**

(b)A large car hire firm has kept records of the cars it owned in 1997-98. All its cars are sold when they become exactly 4yrs unless they are previously damaged in an accident or suffer mechanical failure, in which events they are disposed off. The firm also buys and sells cars for other reasons at any age under 4yrs

Let α = accidental damage, β = mechanical failure

Curtate age of car x	No of car cars in fleet on	No of car cars in fleet on	No of car cars in fleet on	$\theta_x^{\alpha} = NS$ of cars in an accident age x last birthday	$\theta_x^{\beta} = NS$ of cars which breakdown at age x last birthday
0	560	502	486	23	11
1	506	492	500	19	15
2	451	511	480	15	28
3	309	389	372	20	56

(i)Estimate the dependent probabilities of exit by

(a) Accident at each age x

(b) Breakdown at each age x

(ii)Estimate the central rates of decrement by these modes at each age x **[17 Marks]**

Question Two

(a)Let α and β be the model of decrement in a double decrement table. Suppose that α is uniformly distributed over the year of x to x+1 in its associated single decrement table and $\mu_{x+t}^{\alpha} = c$ for $0 \leq t \geq$ 1find formulae foraq_x^{α} andaq_x^{β} in terms q_x^{α} and q_x^{β}

(b)For a certain group of Married women, for whom remarriage is not permitted, the dependent q-type widowhood at each integer age x from 50 to52 inclusive is twice the corresponding dependent q-type rate of mortality. The dependent rates of mortality of wives follow A1967-70 ultimate females

(i) Using a radix of 100,000 and assuming a uniform distribution of each mode of decrement in its associated single decrement table, construct a double decrement table for married women from age 50 to52 inclusive, giving also the value of $(al)_{53}$, the number of wives at age 53

(ii) Find the probability that a wife now aged 50 will be
   (a) Be a live and married at age 53
   (b) Be widowed within 3years **[20 Marks]**

Question Three

Employees of a certain company are given the opportunity of early retirement immediately after they complete a 3yr overseas assignment. Those who undertake this assignment effect a 3yr policy providing the following benefits payable at the end of the year of claim

(a) On death or ill-health retirement during the term, the sum of £6000o
(b) On survival as an employee of the company the end of the term, a lump sum
(c) On withdrawal from the company during the second year of an amount equal $1\frac{1}{4}$ times the annual premiums and on withdrawal from the company during the third year an amount equal to $2\frac{1}{2}$ times the annual premium. No benefit is payable on withdrawal from the company in the first year

The multiple decrement tables is applicable to employees going overseas at exact age 47years.

D =death, W= withdrawal, i= ill health retirement

x				
47	100000	853	13,059	23,007
48	63,081	616	11,604	8,468
49	42,393	478	9,035	1,875
50	31,005			

Using an interest rate of 4%pa, calculate the lump sum for an employee going overseas at exact ages 47 who pays level annual premium of 2000 in advance, ignore expenses

[20Marks]

Question Four

(i)Given that ${}_np_x = 0.3,\quad {}_np_y = 0.4,\quad {}_np_z = 0.6$ Find the probability that of the x,y and z

(a) None will survive n years
(b) Exactly one will survive n years
(c) At least one will survive n years

(ii)Express in terms of p_x , p_y , p_z the probabilities that of the three lives x, y, z

(a)All will survive one year
(b)Atleast one will survive one year
(c)Exactly two will survive one year
(d)Atleast two will survive one year**[20 Marks]**

Question Five

Define in words $1000\overline{A}^{1}_{30:40}$

(i) Calculate:
   (a) $1000\overline{A}^{1}_{30:40}$
   (b) The annual premium payable continuously until the 2nd death for the above assurance in (a) with a sum assured of $1000
   Basis: μ=0.02 for a life aged 30 exact at entry level throughout their life
   μ =0.03 for a life aged 40 exact at entry level throughout their life
   δ= 0.05 throughout
   Expenses: Nil

(ii) Outline the main deficiency of the above premium paying scheme and suggest an alternative **[20 Marks]**