**SAS305: STOCHASTIC PROCESSES**

**INSTRUCTION : ANSWER QUESTION ONE AND ANY OTHER**

**Question ONE : Compulsory** **[30 marks]**

**(I) (a)** define each of the following examples of a stochastic process

(b) a symmetric simple random walk

(c) a compound Poisson process

[4marks]

(II) For each of the processes in (i), classify it as a stochastic process according toits state space and the time that it operates on. [3 Marks]

(III) In the context of a stochastic process denoted by $\{X_t : t \in j\}$, define:

(a) State space

(b) Time set

(c) Sample path [6marks]

(IV) Stochastic process models can be placed in one of four categories according towhether the state space is continuous or discrete, and whether the time set iscontinuous or discrete. For each of the four categories:

(a) State a stochastic process model of that type.

(b) Give an example of a problem an actuary may wish to study using amodel from that category.

[4marks]

**(b)** You have been commissioned to develop a model to project the assets and liabilities of an insurer after one year. This has been requested following a change in the regulatory capital requirement. Sufficient capital must now be held such that there is less than a 0.5% chance of liabilities exceeding assets after one year.

The company does not have any existing stochastic models, but estimates have been made in the planning process of worst case scenarios.

Set out the steps you would take in the development of the model. [13marks]

**Question Two**

Marital status is considered using the following time-homogeneous, continuous time

Markov jump process:

- the transition rate from unmarried to married is 0.1 per annum
- the divorce rate is equivalent to a transition rate of 0.05 per annum
- the mortality rate for any individual is equivalent to a transition rate of 0.025 perannum, independent of marital status

The state space of the process consists of five states: Never Married (NM),

Married (M), Widowed (W), Divorced (DIV) and Dead (D).$p_x$is the probability that a person currently in state $x$, and who has never previouslybeen widowed, will die without ever being widowed.

(i) Construct a transition diagram between the five states. **[4]**

(ii) Show, by general reasoning or otherwise, that $P_{NM}$equals$P_{DIV}$. **[4]**

(iii) Demonstrate that $P_{NM} = \frac{1}{5} + \frac{4}{5} P_M$ and $P_M = \frac{1}{4} + \frac{1}{2} P_{DIV}$**[3]**

(iv)Calculate the probability of never being widowed if currently in state NM. **[5]**

(v) Suggest two ways in which the model could be made more realistic. **[4]**

**[Total 20 Marks]**

**Question Three**

A No-Claims Discount system operated by a motor insurer has the following four levels:

Level 1: 0% discount

Level 2: 25% discount

Level 3: 40% discount

Level 4: 60% discount

The rules for moving between these levels are as follows:

Following a year with no claims, move to the next higher level, or remain atlevel 4.

Following a year with one claim, move to the next lower level, or remain atlevel 1.

Following a year with two or more claims, move back two levels, or move tolevel 1 (from level 2) or remain at level 1.

For a given policyholder the probability of no claims in a given year is 0.85 and theprobability of making one claim is 0.12.

$X(t)$ denotes the level of the policyholder in year $t$.

(i) (a) Explain why $X(t)$ is a Markov chain. [3]

(b) Write down the transition matrix of this chain.[3]

(ii) Calculate the probability that a policyholder who is currently at level 2 will beat level 2 after:

(a) one year

(b) two years

(c) three years[6]

(iii) Explain whether the chain is irreducible and/or aperiodic.[4]

(iv) Calculate the long-run probability that a policyholder is in discount level 2. [4]

**[Total 20 Marks]**

## Question Four

An insurance policy covers the repair of a washing machine, and is subject to a maximum of 3 claims over the year of coverage.

The probability of the machine breaking down has been estimated to follow anexponential distribution with the following annualised frequencies, $\mu$

$\mu$ =1/10 If the machine has not suffered any previous breakdown.

$\mu$= 1/5 If the machine has broken down once previously.

$\mu$=1/4 if the machine has broken down on two or more occasions.

As soon as a breakdown occurs an engineer is dispatched. It can be assumed that the repair is made immediately, and that it is always possible to repair the machine.

The washing machine has never broken down at the start of the year (time $t$ = 0).

$P_i(t)$Is the probability that the machine has suffered $i^{th}$ breakdowns by time $t$.

(i) Draw a transition diagram for the process defined by the number ofbreakdowns occurring up to time $t$. [1]

(ii) Write down the Kolmogorov equations obeyed by $p_0'(t), p_1'(t)$ and $p_2'(t)$ . [2]

(iii) (a) Derive an expression for $p_0(t)$ and

(b) Demonstrate that $P_1(t) = e^{-\frac{t}{10}} - e^{-\frac{t}{5}}$

(iv) Derive an expression for $P_2$(t)

(v) Calculate the expected number of claims under the policy.**[20MARKS]**

**Question Five**

A motor insurance company wishes to estimate the proportion of policyholders who make at least one claim within a year. From historical data, the company believes that the probability a policyholder makes a claim in any given year depends on the number of claims the policyholder made in the previous two years. In particular: the probability that a policyholder who had claims in both previous years will make a claim in the current year is 0.25the probability that a policyholder who had claims in one of the previous two years will make a claim in the current year is 0.15; and the probability that a policyholder who had no claims in the previous two years will make a claim in the current year is 0.1

(i) Construct this as a Markov chain model, identifying clearly the states of the chain.

(ii) Write down the transition matrix of the chain.

(iii) Explain why this Markov chain will converge to a stationary distribution.

(iv) Calculate the proportion of policyholders who, in the long run, make at least one claim at a given year.

**[Total 20 MARKS]**