JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

DRAFT-UNIVERSITY EXAMINATIONS
2012/2013 ACADEMIC YEAR

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCES

SEMESTER ONE, SECOND YEAR EXAMINATIONS for BSc/BEd

SMA201: LINEAR ALGEBRA II

Aug,2013 Time: 2hrs

INSTRUCTIONS

Answer **Question1** and **two** other questions

Show all the necessary working

Question1 [30marks] Compulsory

(a) Define a linear mapping T from an m-dimensional vector space X into n-dimensional vector space Y over the real field F. Let $B=\{u_1,u_2,u_3,.........,u_m\}$, $\Psi=\{v_1,v_2,v_3,.........,v_n\}$ be the bases for the respective vector spaces X and Y.Construct matrix M of T with respect to the ordered bases $B,\ \Psi$. [4marks]

(b) If $A=\begin{bmatrix}1-4i & i & 2\\ 3 & 2+i & 0\end{bmatrix}$

Find the matrix A^* the ***adjoint*** of **A**. [5marks]

(c) Let $P_{n\times n}$ be a real square matrix.

(i)Define what is meant by $P_{n\times n}$ is orthogonal.

(ii) Let $P=\begin{pmatrix}1 & -2 & 2\\ 2 & 2 & 1\\ -2 & 1 & 2\end{pmatrix}$ be a real square matrix.

Compute PP^t, P^tP, P^{-1} and orthonormalize P. **[8marks]**

(d) If V is a complex space with a norm that satisfies $\langle x,y\rangle_r=\dfrac{\|x+y\|^2-\|x-y\|^2}{4}$

prove that $\langle x,y\rangle_r=\dfrac{\|x+y\|^2-\|x-y\|^2}{4}$ *is an inner product on V.* **[7marks]**

(e) Determine the Fourier expansion of $x=\begin{pmatrix}-1\\ 2\\ 1\end{pmatrix}$ with respect to the vectors **[6marks]**

Question2 [20marks]

Given the vectors $x_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}, x_2 = \begin{pmatrix} 1 \\ 2 \\ 0 \\ -1 \end{pmatrix}, x_3 = \begin{pmatrix} 3 \\ 1 \\ 1 \\ -1 \end{pmatrix}$ of vector space R^4 with the standard inner product.

(i) Show that $x_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}, x_2 = \begin{pmatrix} 1 \\ 2 \\ 0 \\ -1 \end{pmatrix}, x_3 = \begin{pmatrix} 3 \\ 1 \\ 1 \\ -1 \end{pmatrix}$ are linearly independent.

(ii) Show that the vectors $x_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}, x_2 = \begin{pmatrix} 1 \\ 2 \\ 0 \\ -1 \end{pmatrix}, x_3 = \begin{pmatrix} 3 \\ 1 \\ 1 \\ -1 \end{pmatrix}$ are mutually non orthogonal.

(iii) Apply the Gram-Schmidt process to the vectors $x_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}, x_2 = \begin{pmatrix} 1 \\ 2 \\ 0 \\ -1 \end{pmatrix}, x_3 = \begin{pmatrix} 3 \\ 1 \\ 1 \\ -1 \end{pmatrix}$

to obtain the corresponding orthogonal set of vectors $a_1, \quad a_2, \quad a_3$ given .

(iii)Verify that the set $\{a_1, \quad a_2, \quad a_3\}$ linearly independent. **[20marks]**

Question3 [20marks]

Let W be the space of all 3×3 matrices A over R which are skew- symmetric *i.e.,* $A^t = -A$. We equip W with the inner product $[A * B] = tr[AB^t]$. Let V be the vector space R^3 with the standard inner product. If T be the mapping from V into W defined by

$$T(x, y, z) = \begin{bmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{bmatrix} \quad i.e. \; T: V \to W$$

(a)Show that $T(0_V) = 0_W$ **[5marks]**

(b)Show that T is linear **[7marks]**

(c) Prove that T is a vector space isomorphism. **[8marks]**

Question4 [20marks]

Let $A=\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

. Find

(i)The characteristic polynomial of A. **[6marks]**

(ii) The eigenvalues of A. **[6marks]**

(iii)The corresponding eigenvectors. **[8marks]**

Question5 [20marks]

Given P_1, P_2, P_3 are rotator matrix maps that corresponds to

$$P_1=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}, P_2=\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 & 1 \\ 0 & \sqrt{2} & 0 \\ -1 & 0 & 1 \end{bmatrix}, P_3=\begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 \\ \sqrt{3}/2 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Prove that

(i) rotator matrix maps P_i are orthogonal for i=1,2,3.

(ii) rotator matrix map products $P_i * P_j$ are orthogonal for i, j=1,2,3.

(iii) If $P = P_i * P_j$ determine the inverse matrix P^{-1} **[20marks]**