SMA 300 REAL ANALYSIS I

Answer Question ONE and any other TWO questions

SECTION A(30 MARKS) Answer both questions

QUESTION ONE (30MARKS)

(a) Define the following terms.

(i) A continuous function at a point x_o .

(ii) Refinement of a partition P

(4MARKS)

(b)(i)Determine whether or not $x_n = n^2$ is a fundamental sequence.

(3MARKS)

(ii) Determine the convergence of the infinite series;

$$\sum \frac{1}{n^2 + a^2}$$

(4MARKS)

(c) Prove that the sequence $(\frac{2n-7}{3n+2})$ is;

(i) Monotonically increasing.

(3MARKS)

(ii) Bounded

(3MARKS)

(d)(i) Prove that a finite intersection of open sets in a metric space is open.

(4MARKS)

(ii) Show that the function $f(x) = 2x+1$ is Riemann integrable in the interval $[1, 3]$ and that $\int_1^3 f(x)dx = 10$.

(5MARKS)

(iii) Use an example to demonstrate that not all bounded sequences are convergent.

(4MARKS)

QUESTION TWO(20MARKS)

(a)Prove that a countable union of countable set is countable.

(5MARKS)

(b)(i) Prove that if a sequence x_n converges to a then every subsequence of x_n also converges to a.

(3MARKS)

(ii) State any two properties of limits.

(2MARKS)

(iii) Test for the convergence of the following series $1 + \frac{x^2}{2} + \frac{x^4}{4} + \frac{x^6}{6} + ...$

(6MARKS)

(c) Prove that every Cauchy sequence is bounded.

(4MARKS)

QUESTION THREE(20MARKS)

(a)(i) State Cauchy's integral test for convergence

(2MARKS)

(ii) By using Cauchy's integral test, show that $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p > 1$ and diverges if $p \leq 1$

(6MARKS)

(b)(i) Prove that the set $\mathbb{R}$ with the mapping $d : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ defined as $d(x, y) =\mid x - y \mid$ is a metric space

(5MARKS)

(ii) Prove that every monotonically increasing sequence which is bounded converges.

(7MARKS)

QUESTION FOUR(20MARKS)

(a)(i)State without proof the De-Alemberts ratio test for convergence of the series $\sum x_k$.

(3MARKS)

(ii)Show that the function $f(x) = x^2$ is Riemann integrable in the interval $[0, k]$, hence find the integral(k is a constant)

(10MARKS)

(b)Prove that the set of real numbers is uncountable.

(7MARKS)

QUESTION FIVE(20MARKS)

(a) Show that a function $f(x) = \frac{1}{x}$ is continuous but not uniformly continuous for all $x \in (0, 1)$

(7MARKS)

(b) Show that $\sum(-1)^n \frac{n+2}{2^n+5} \cos nx$ is convergent for all real values of x.

(6MARKS)

(c)(i) Define an interior point of a set.

(2MARKS)

(ii) Prove that the interior of a set is an open set.

(5MARKS)