JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY SCHOOL OF MATHEMATICS

UNIVERSITY EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE LIBRARYINFORMATION SCIENCE
$1^{\text {ST }}$ YEAR $1^{\text {ST }}$ SEMESTER 2013/2014 ACADEMIC YEAR
CENTRE: KISUMU

COURSE CODE: SMA 3113
COURSE TITLE: LOGICAL FUNCIONS

EXAM VENUE:

DATE: 5/12/2013
TIME: 2 HOURS

## Instructions:

1. Answer question 1 (Compulsory) and ANY other 2 questions
2. Candidates are advised not to write on the question paper.
3. Candidates must hand in their answer booklets to the invigilator while in the examination room.

## QUESTION ONE (30 marks)

a) Give precise definitions of the following terms:
(i) Proposition (ii) Contrapositive (iii) Tautology
[3 marks]
b) Find the bitwise OR, bitwise AND, and bitwise XOR of the following pairs of bit strings: 1001100110 and 1111000011
[3 marks]
c) Let $p$ and $q$ be the propositions:
$p:$ I bought a lottery ticket this week
$q$ : I won the million jack point on Friday.
Express each of the following as an English sentence.
(i) $\tau p \rightarrow \tau q$ (ii) $\tau p \vee(p \wedge q)$ (iii) $p \wedge(\tau p \vee q)$
d) Determine whether $(\tau q \wedge(p \rightarrow q) \rightarrow \tau q$ is a tautology.
e) Let $Q(x, y)$ denote the proposition: " $x+y$ " $=0$. What are the truth Values of (i) $\exists y \forall x Q(x, y) \quad$ (ii) $\forall x \exists y Q(x, y)$.
f) Determine whether $p \leftrightarrow q$ and $\tau p \vee q$ are logically equivalent or not.
g) Show that the sets $A-(B \cap C)$ and $(A-B) \cap(A-C)$ are equal
h) Find the power set $P(s)$ if $s=\{1,2, a, b, c, d\}$
i) Let $A=\{2,4,6\}$ and $B=\{1,3,5,7\}$ find $A \times B$

## QUESTION TWO (20 marks)

a) Derive an algorithm that finds the sum of all the integers in the list [7marks]
b) Find the prime factorization of 7007
[6 marks]
c) Use mathematical induction to show that
$1^{2}+2^{2}+\ldots+n^{2}=n(n+1)(2 n+1) / 6$
[7 marks]

## QUESTION THREE (20 marks)

a) Find the base 8 expansion of $(12345)_{10}$
b) Decrypt the message: HDWGLPVXP using Caesar's Cipher.
c) What are the solutions of the linear congruence $3 x=4(\bmod 7)$.

## QUESTION FOUR (20 marks)

Each user of a computer system has a password which is six to eight characters long, where each character is an uppercase letter or a digit. Each password must contain at least one digit. How many possible passwords are there?
[20 marks]

## QUESTION FIVE (20 marks)

a) Distinguish between a floor function and a ceiling function and hence evaluate: (i) $\lfloor-2.3\rfloor$
(ii) $\lceil-12.2\rceil$
[4 marks]
b) Let $A, B$, and $C$ be sets. Show that

$$
\overline{A \cup(B \cap C)}=(\bar{C} \cup \bar{B}) \cap \bar{A}
$$

c) Assume that in a group of six people, each pair of individuals consists of two friends or two enemies. Show that there are either three mutual friends or there are three mutual enemies in the group.

