JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

DRAFT - EXAMINATIONS 2012/2013

REGULAR

SEMESTER 1 FIRST YEAR IV EXAMS

COURSE CODE: SMA 405

COURSE TITLE: PARTIAL DIFFERENTIAL EQUATION I

DATE : Aug, 2013 ***TIME: 2hrs***

INSTRUCTIONS

ATTEMPT **question1** and **two** other QUESTIONS

Show all the necessary working

Question 1[30 marks] Compulsory

(a) Given the partial differential equation

(i) $\left(x-F\right)^3\frac{\partial F}{\partial x}-\left(F^2+t\right)\frac{\partial F}{\partial t}=3xt^2$ (ii) $x^2\frac{\partial^2 F}{\partial x^2}-y^2\frac{\partial^{12}F}{\partial y^{12}}+x\frac{\partial F}{\partial x}-y\frac{\partial F}{\partial x}\frac{\partial F}{\partial y}=0$

(iii) $x^2\frac{\partial^5 F}{\partial x^5}-y^2\left(\frac{\partial^2 F}{\partial y^2}\right)^{14}+x\frac{\partial F}{\partial x}-y\frac{\partial F}{\partial y}=0$

State in each case, the order, degree and whether linear or nonlinear. [10marks]

(b) Consider the second order linear partial differential equation

$$y\frac{\partial^2 u}{\partial x^2}+6\frac{\partial^2 u}{\partial x\partial y}+4\frac{\partial^2 u}{\partial y^2}=0,$$

(i) Classify the partial differential equation

(ii) Obtain the characteristic equation of the partial differential equation

(i) Solve the partial differential equation. [10 marks]

(c) Consider the second order linear partial differential equation

$$a\frac{\partial^2 u}{\partial x^2}+b\frac{\partial^2 u}{\partial x\partial y}+c\frac{\partial^2 u}{\partial y^2}+d\frac{\partial u}{\partial x}+e\frac{\partial u}{\partial y}+fu+g=0, \quad :u\left(x,y\right)$$

where a, b, c, d, e, f, g are in general variable coefficients which may depend on real x or y with $u\left(x,y\right)$ as the dependent variable. Use discriminant $\Delta\left(a,b,c\right)$ theory to categorize; elliptic, parabolic and hyperbolic partial differential equations ;

(i) $\frac{\partial^2 u}{\partial x^2}+4x^2y^{14}\frac{\partial^2 u}{\partial y^2}=1$ (ii) $\frac{\partial u}{\partial t}=121t^6\frac{\partial^2 u}{\partial x^2}$

(iii) $\frac{\partial^2 u}{\partial x^2}+x^3\frac{\partial^2 u}{\partial y^2}=0$ (iv) $\frac{\partial^2 u}{\partial t^2}-t^2x^{12}\frac{\partial^2 u}{\partial x^2}=10t$. [10 marks]

(d) Solve the first order partial differential equation $\frac{\partial z}{\partial x} z - \frac{\partial z}{\partial y} z = z^2 + 2(x+y)^2$ [5marks]

(e) Determine the function $z(x, y)$ which satisfies the linear second order partial differential equation $\left(D^2 - DD' - 6D'^2\right) z = 0$ [6marks]

Question 2 [20marks]

Given the function $F(x, y) = 4x^2 y - y^2 - 8x^2 - 2x^4 + 4000$

(i) Find $\frac{\partial F}{\partial x}$, $\frac{\partial F}{\partial y}$, [4 marks]

(ii) Find $\frac{\partial^2 F}{\partial x^2}$, $\frac{\partial^2 F}{\partial y^2}$ and $\frac{\partial^2 F}{\partial x \partial y}$ [5marks]

(iii) Determine and distinguish all the stationary points of F [11 marks]

Question 3[20marks]

(a) Eliminate the arbitrary functions f, g from the equation

$$u = f(x+y) + g(x-y) + \frac{1}{144} x(x-y)^2$$ [6marks]

(b) Solve the linear second order partial differential equations

(ii) $\left(4D^2 - 12DD' + 9D'^2\right) z = 0$ [6marks]

(iii) $\left(D^3 - 3D^2 D' - 4D'^3\right) u = e^{18x+2y}$ [8marks]

Question 4 [20marks]

(a) Use characteristic method to solve the linear partial differential equation
$u_x + u_y = 2$
subject to the initial condition $u(x, 0) = x^2$. [10 marks]
(b) Eliminate the arbitrary functions f, g, h from the equation

(i) $u = f(x - 2t + iby) + g(x - 2t - iby)$ $: i = \sqrt{-1}$

(ii) $u = f\left(x+y\right)+g\left(x-y\right)+h\left(2x+y\right)-\frac{1}{2}x\left(x-y\right)^2 e^{x+y}$ [10 marks]

Question 5[20marks]

Solve the initial boundary value heat equation

$u_t = u_{xx}, \quad 0<x<1, t>0$

satisfying the conditions

$u\left(0,t\right)=1, \; u\left(1,t\right)=1 \quad 0<x<1, t>0$

$u\left(x,0\right)=1+\cos 2\text{®}x, \quad 0<x<1$ [20marks]