Jaramogi Oginga Odinga University of Science and Technology

UNIVERSITY *DRAFT* EXAMS 2012/2013

SCHOOL OF INFORMATICS

SEMESTER ONE, FIRST YEAR BSC EXAMINATIONS

SMA3114: ANALYTICAL COMPUTATIONAL METHODS

Aug 2013 Time 2hours

Instructions

Answer Question1 and TWO other questions.

Show all the necessary working

Question1 [30 marks] COMPULSORY

(a)E valuate (i) $\lim\limits_{x \to 2}\left\{\dfrac{x-4}{x+4}\right\}$ (ii) $\lim\limits_{x \to -1}\left\{\dfrac{x^2-1}{x-1}\right\}$ (iii) $\lim\limits_{h \to 0}\left\{\dfrac{(x+h)^2-(x-h)^2}{2h}\right\}$ [4 marks]

(b) Using primitive definition of derivative, determine

$\dfrac{dy}{dx}$ *given*, $y(x) = x^2 - 2x - 4$

(c) (i) Solve sin θ = $\frac{1}{2}$ for $0° \leq \bar{\ } \leq 720°$ [5 marks]

(ii)Find x if $\sqrt{3}\sin x - \cos x = 1$: $0° \leq x \leq 360°$ [5 marks]

(d) Evaluate the indefinite integral (i) $\int 4x^2 dx$ (ii) $\int\left(\dfrac{1}{x^2} + \cos x\right)dx$ (iii) $\int \dfrac{2+4x}{x+x^2}dx$ [6 marks]

.(e) If $Z = 2 + 2i$ express the complex numbers $\bar{Z}$, $\bar{Z}Z$ in the form $a + ib$. [5 marks]

(f) The distance S in meters, of a particle traveled from a point Q after time t seconds, is given by $S = \dfrac{t^3}{3} - 144t + 210$. When is the particle instantaneously at rest? Determine the speed and acceleration of the particle after12 seconds. [5 marks]

Question2 [20 marks]

(a) (i) Prove that $\dfrac{d}{dx}\left(u(x)v(x)\right) = v(x)\dfrac{du(x)}{dx} + u(x)\dfrac{dv(x)}{dx}$ [4 marks]

(ii) If $y = \left(3 + x^2\right)e^x$ find $\dfrac{dy}{dx}$, at $x = 0$ [5 marks]

(b) (i) Given $y = \log_e\left(3 + 4x^2\right)$ find $\dfrac{dy}{dx}, \dfrac{d^2y}{dx^2}$, at $x = 1$ [6 marks]

(c) Determine $\dfrac{dy}{dx}$, if $y = \left[\dfrac{3+x^2}{e^{\sim x}}\right]^{-1}$ [5 marks]

Question3 [20 marks]

Define a curve $y(x) = x^3 - 48x + 25$.

(i)Find $\dfrac{dy}{dx}, \dfrac{d^2y}{dx^2}$ (ii) Determine and distinguish the turning points of the curve.

(iii) Sketch the curve $y = x^3 - 48x\ ; -150 \leq x \leq 150$ [15 marks]

(iv) State both the minimum and maximum values of the $y = x^3 - 48x\ ; -150 \leq x \leq 150$ [5 marks]

Question4 [20 marks]

.(a.) If $Z_1 = \frac{1}{2} + \frac{1}{2}i$, $Z_2 = 2 - 2i$ express the complex numbers , Z_1^{10}, Z_2^{10} in the form $Z = r\left(\cos^{-} + i\sin^{-}\right)$. [6 marks]

(b) On same Argand plane, plot the complex numbers Z_1 , Z_1^{10}, Z_2 , Z_2^{10}.
Determine the effect of raising a complex number to a positive power. [6 marks]

(c) Find x if $\sqrt{3}\sin x - \cos x = 1$: $0° \le x \le 720°$ [8 marks]

Question5 [20 marks]

(a) Given the matrices $A = \begin{bmatrix} 1 & -1 & 2 \\ -2 & 1 & 1 \\ -1 & 2 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} a \\ b \\ c \end{bmatrix}, C = \begin{bmatrix} 16 \\ 0 \\ 0 \end{bmatrix}$

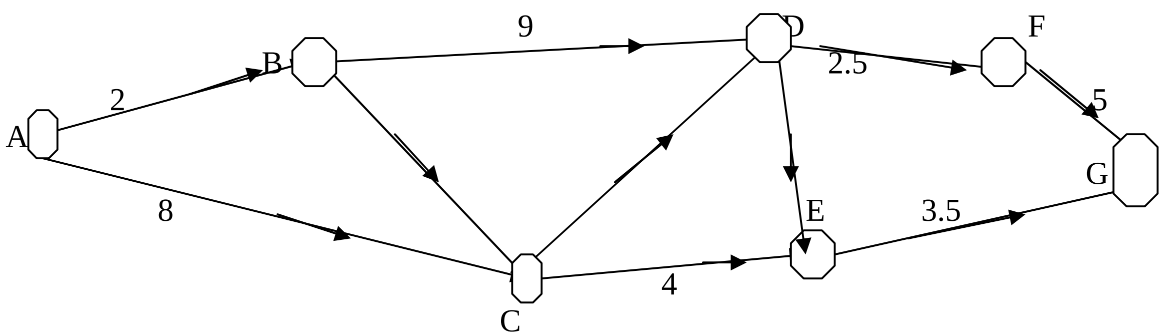
(i). Compute det A

(ii). Show that det $A \neq 0$

(iii) If $\begin{bmatrix} 1 & -1 & 2 \\ -2 & 1 & 1 \\ -1 & 2 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{5}{2} & -\frac{7}{2} \\ \frac{1}{2} & \frac{3}{2} & -\frac{5}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{5}{2} & -\frac{7}{2} \\ \frac{1}{2} & \frac{3}{2} & -\frac{5}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 \\ -2 & 1 & 1 \\ -1 & 2 & 1 \end{bmatrix} = P$, find matrix P and give the inverse of A.

(iv) Using the results of part (iii) above solve the equation $AX = C$. [15 marks]

(c) Find the shortest path from source to sink on the system below.



[2 marks]

(d) Determine the constant term in the binomial expansion $\left(x - \frac{5}{x}\right)^{10}$ [3 marks]