JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

3^{rd} YEAR 2^{nd} SEMESTER EXAMINATION [**SCHOOL BASED**]

SMA 304: GROUP THEORY I

INSTRUCTION: Attempt question one (**COMPULSORY**) and any other TWO questions only.

QUESTION ONE(*COMPULSORY*) [30 MARKS]

(a). Show that any group G of prime order p is cyclic and has no nontrivial subgroup. (5 marks)

(b). Differentiate between a monoid and a semigroup. (4 marks)

(c). Show that if b is an element of a group G of order k, then $b^k = e$, where e is the identity element in G. (4 marks)

(d). Show that the intersection of any nonempty collection of subgroups of a group G is also a subgroup of G. (4 marks)

(e). State and prove the subgroup criterion. (5 marks)

(f). Define a normalizer of a group G and show that it is a subgroup of G. (4 marks)

(g). Define a binary operation and Quotient group. (4 marks)

QUESTION TWO [20 MARKS]

(a). State and prove the first isomorphism theorem for groups. (15 marks)

(b). Prove that if H and K are subgroups of G with N normal in G, then $H/(H \cap K)$ is isomorphic to HK/K. (5 marks)

QUESTION THREE [20 MARKS]

(a). Define a normal group. (2 marks)

(b). Prove that every subgroup of a commutative group is a normal group. (5 marks)

(**c**). Prove that the centre of a group G is a normal subgroup of G.(5 marks)

(d). State and prove the Quotient group theorem. (8 marks)

QUESTION FOUR [20 MARKS]

(a). Distinguish between even permutation and odd permutation.(4 marks)

(b). Prove that the number of elements in the symmetric group S_k is $k!$. (10 marks)

(c). Describe permutation of a set, symmetric group and cyclic group. (6 marks)

QUESTION FIVE [20 MARKS]

(a). Explain the meaning of an index of a group and order of a group. (4 marks)

(b). State and prove the following theorems for groups:

(i). Lagrange's Theorem. (9 marks)

(ii). Fermat's Theorem. (7 marks)