JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE & TECHNOLOGY UNIVERSITY EXAMINATIONS 2012/2013

4TH YEAR 1ST SEMESTER EXAMINATION OF BACHELOR OF EDUCATION (SCIENCE)

MAIN SCHOOL BASED

COURSE CODE: SMA 405

COURSE TITLE: PARTIAL DIFFERENTIAL EQUATION I

DATE: 27 / 8 /2013 **TIME: 2 .00 - 4.00 PM**

DURATION: 2 HOURS

INSTRUCTIONS

1. **This paper contains five (5) questions.**
2. **Answer question 1 (compulsory) and ANY other TWO questions.**
3. **Write all answer in the booklet provided.**

Show all the necessary working

Question 1[30 marks] Compulsory

(a) Given the partial differential equation

(i) $x\frac{\partial F}{\partial x}-y\frac{\partial F}{\partial y}=3xy^2$ (ii) $x^2\frac{\partial^2 F}{\partial x^2}-y^2\frac{\partial^2 F}{\partial y^2}+x\frac{\partial F}{\partial x}-y\frac{\partial F}{\partial x}\frac{\partial F}{\partial y}=0$

(iii) $x^2\frac{\partial^3 F}{\partial x^3}-y^2\left(\frac{\partial^2 F}{\partial y^2}\right)^4+x\frac{\partial F}{\partial x}-y\frac{\partial F}{\partial y}=0$

State in each case, the order, degree and whether linear or nonlinear. [7marks]

(b) Consider the second order linear partial differential equation

$$\frac{\partial^2 u}{\partial x^2}+2\frac{\partial^2 u}{\partial x\partial y}+x\frac{\partial^2 u}{\partial y^2}=0,$$

(i) Classify the partial differential equation

(ii) Obtain the characteristic equation of the partial differential equation

(i) Solve the partial differential equation. [5marks]

(c) Consider the second order linear partial differential equation

$$a\frac{\partial^2 u}{\partial x^2}+b\frac{\partial^2 u}{\partial x\partial y}+c\frac{\partial^2 u}{\partial y^2}+d\frac{\partial u}{\partial x}+e\frac{\partial u}{\partial y}+fu+g=0, \quad :u(x,y)$$

where a, b, c, d, e, f, g are in general variable coefficients which may depend on real x or y with $u(x,y)$ as the dependent variable. Use discriminant $\Delta(a,b,c)$ theory to categorize; elliptic, parabolic and hyperbolic partial differential equations ;

(i) $\frac{\partial^2 u}{\partial x^2}+4\frac{\partial^2 u}{\partial y^2}=1$ (ii) $\frac{\partial u}{\partial t}=t^6\frac{\partial^2 u}{\partial x^2}$ (iii) $\frac{\partial^2 u}{\partial x^2}+3\frac{\partial^2 u}{\partial y^2}=0$ (iv) $\frac{\partial^2 u}{\partial t^2}-\frac{\partial^2 u}{\partial x^2}=0$. [10 marks]

(d) Solve the first order partial differential equation $\frac{\partial z}{\partial x}z-\frac{\partial z}{\partial y}z=z^2+(x+y)^2$ [2marks]

(e) Determine the function $z(x, y)$ which satisfies the linear second order partial differential

equation $\left(D^2 + DD' - 6D'^2\right)z = 0$ [6marks]

Question 2 [20marks]

Given the function $F(x, y, z) = 8x^2 + 24y^2 + 16z^2 + 24x + 16z + 1$,

(i) Find $\dfrac{\partial F}{\partial x}$, $\dfrac{\partial F}{\partial y}$, [4 marks]

(ii) Find $\dfrac{\partial^2 F}{\partial x^2}$, $\dfrac{\partial^2 F}{\partial y^2}$ and $\dfrac{\partial^2 F}{\partial x \partial y}$ [5marks]

(iii) Determine and distinguish all the stationary points of F [11 marks]

Question 3[20marks]

(a) Eliminate the arbitrary functions f , g from the equation

$$u = f(x+y) + g(x-y) + \frac{1}{4}x(x-y)^2$$

State the order and degree of the resulting differential equation. [11marks]

(b) Solve the linear second order partial differential equations

$(4D^2 - 12DD' + 9D'^2) = ¢(x-2, y-1)$ [9marks]

Question 4 [20marks]

(a) Use characteristic method to solve the linear partial differential equation

$u_x + u_y + u = 1$, subject to the initial condition $u = \sin x$, on $y = x + x^2$, $x > 0$.[14 marks]

(b) Eliminate the arbitrary functions f , g from the equation

$u = f(x - at + iby) + g(x - at - iby) \quad : i = \sqrt{-1}$ **[6 marks]**

Question 5[20marks]

Solve the initial boundary value heat equation

$u_t = u_{xx}$, $\quad 0 < x < 1$, $t > 0$

satisfying the conditions

$u(0,t) = 100$, $u(1,t) = 100 \quad 0 < x < 1$, $t > 0$

$u(x,0) = 1 + \sin \text{®} x$, $\quad 0 < x < 1$ **[20 marks]**