JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE & TECHNOLOGY UNIVERSITY EXAMINATIONS 2012/2013

1ST YEAR 1ST SEMESTER EXAMINATION OF BACHELOR OF EDUCATION (SCIENCE)

MAIN SCHOOL BASED

COURSE CODE: SMA 103

COURSE TITLE: LINEAR ALGEBRA I

DATE: / /2013 **TIME: .00 - .00**

DURATION: 2 HOURS

INSTRUCTIONS

1. **This paper contains five (5) questions.**
2. **Answer question 1 (compulsory) and ANY other TWO questions.**
3. **Write all answer in the booklet provided.**

Show all the necessary working

Question1 [30marks] Compulsory

(a) Define the vector subspaces H_1, H_2 of vector space R^3 by , $H_1 = \{(x, y, z) : x + 2y + 2z = 0 \}$,
$H_2 = \{(x, y, z) : 2x + 2y - 8z = 0\}$.

(i) Verify that both H_1, H_2 do contain the zero vector. [2 marks]

(ii) Find bases for H_1, H_2. [4 marks]

(b) Suppose the mapping $L : R^3 \to R^3$ with $L\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x - y - z \\ x + y + z \\ z \end{bmatrix}$

(i) Show that L is linear. (ii)Determine ker (L) and Im(L). [8 marks]

(c) Given the system of linear equations

$2x + y = 70$

$5x + 3y = 20$

(i) express it in the matrix form $A\underset{\sim}{X} = \underset{\sim}{b}$

(ii) apply the elementary matrix row reduction operations on the associated augmented matrix;

$A : I : \underset{\sim}{b}$ to reduce to the final form $I : \hat{A} : \underset{\sim}{\hat{b}}$ where I is the two by two identity matrix. Compute matrix products $A\hat{A}$, $\hat{A}A$ and hence obtain A^{-1} and $\underset{\sim}{X}$. [9 marks]

(d) Let $P = \begin{bmatrix} 4 & 5 \\ 1 & 1 \end{bmatrix}, R = \begin{bmatrix} 0 & 5 \\ 2 & 1 \end{bmatrix}$

(i) Determine whether or not P, or R are singular. [3 marks]

(ii) Compute the matrices PR , P^{-1}, R^{-1} and show that $[PR]^{-1} = R^{-1}P^{-1}$. [4marks]

Question2 [20marks]

(a) Given matrix $M = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$

(i) Show that $M^2 = 4I_{4\times 4}$ and hence find M^{-1}, the inverse of M .

(ii) Show that the following vectors $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$

are linearly independent. [11 marks]

(b) Suppose $T:[x,y,z] \rightarrow [x, x-y, y]$. Construct matrix A of linear mapping T with respect to an ordered basis for basis for R^3. [9 marks]

Question3 [20marks]

(a) Without using direct computation , show that $\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ are eigenvectors of

the matrix $A = \begin{pmatrix} 1 & -4 & -4 \\ 8 & -11 & -8 \\ -8 & 8 & 5 \end{pmatrix}$. Give the associated eigenvalues a_1, a_2, a_3 of this matrix.

Verify that $trace(A) = a_1 + a_2 + a_3$ [12 marks]

(b) Find the coordinates vector $v_0 = [1,1,5]$ with respect to the ordered basis $\{[1,1,0],[1,2,0],[1,2,1]\}$ [*8* marks]

Question4 [20marks]

Define a linear mapping T from vector space X into vector space Y i.e. $T: X \to Y$

(a) Explain what is meant by (i)kernel of T (ii)image of T (iii)rank of T (iv) nullity of T

[8 marks]

(b) State the relationship between dimension of kernel of T and rank of T [2marks]

(c) For matrix. $M = \begin{pmatrix} 1 & 2 & -2 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$ determine adjoint of M and hence state M^{-1} [10marks]

Question5 [20marks]

Let $A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ -1 & 3 & 4 \end{pmatrix}$ be a matrix of linear transformation T.

(a) Determine kernel of T [6marks]

(b) Determine range of T [4marks]

(c) State nullity and rank of T [4marks]

(d) Determine which of the vectors $[-1,1,-1]$, $[1,0,0]$ belong to the kernel of T. [6marks]