JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

FIRST YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF MASTER OF SCIENCE IN PURE MATHEMATICS

SMA 830: NON COMMUTATIVE RING THEORY

INSTRUCTIONS:

1. This paper consists of FIVE questions

2. Attempt any THREE questions.

3. Observe further instructions on the answer booklet.

QUESTION 1 **[20 Marks]**

(a) Give an example of a $\mathbf{Z}-$ submodule of $(\mathbf{Q}/\mathbf{Z})_{\mathbf{Z}}$ which is Artinian but not Noetherian. **[3 mks]**

(b) Let N be an $R-$ submodule of M. Show that if N and M/N are noetherian, then M is noetherian.

[6 mks]

(c) State and prove Schur's Lemma **[6 mks]**

(d) Demonstrate that even if the ring R is commutative, the right and left module actions of R on itself needn't be the same. **[5 mks]**

QUESTION 2 **[20 Marks]**

(a) State the Artin- Wedderburn theorem **[3 mks]**

(b) Show that if R has increasing chain condition on right ideals, then any nil right or left ideal is nilpotent **[5 mks]**

(c) Let N be the maximal nilpotent ideal of a right artinian ring R. Show that R/N has no nonzero nilpotent ideals **[5 mks]**

(d) i) What is a Dedekind- finite ring? **[1 mk]**
ii) Construct a ring which is not Dedekind finite **[3 mks]**
iii) Demonstrate that a ring which is not Dedekind- finite is neither artinian nor noetherian. **[3 mks]**

QUESTION 3 **[20 Marks]**

(a) Explain the meaning of "Ore Condition" in a ring R. **[3 mks]**

(b) Let R be semiprime right noetherian. Show that every right regular element is also left regular.

[7 mks]

(c) Demonstrate that the quotient ring of a noetherian ring needn't be artinian. **[10 mks]**

QUESTION 4 **[20 Marks]**

(a) Let R be a prime ring with zero divisors. Show that R has nonzero nilpotent elements. **[5 mks]**

(b) With the aid of two examples, describe Brauer Groups **[7 mks]**

(c) Let A and B be central simple over a field F. Show that $A \otimes_F B$ is central simple over F as well **[8 mks]**

QUESTION 5 **[20 Marks]**

(a) Show that the only noncommutative finite dimensional central simple division algebra over $\mathbf{R}$ is the ring of quaternions H.

[11 mks]

(b) Prove the following:

i) A reduced ring is prime iff it is a domain. **[5 mks]**

ii) An algebraic domain is a division ring. **[4 mks]**