JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

UNIVERSITYDRAFT- EXAMINATIONS
2012/2013 ACADEMIC YEAR
SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCES

SEMESTER TWO FIRST YEAR EXAMINATIONS

FOR THE DEGREE
OF
MASTER OF SCIENCE (MSc)

COURSE CODE: SMA 849
COURSE TITLE: NUMERICAL SOLUTION OF PARTIAL DIFFERENTIAL EQUATIONS

DATE : August 2013 TIME: 3hrs

INSTRUCTIONS

Attempt any **three** questions

Show all the necessary working

Question 1 [20 marks]

(a) Categorize the given below equations into; elliptic, parabolic and hyperbolic partial differential equations given below.

(i) $x^2 \frac{\partial^2 u}{\partial x^2} + 144 \frac{\partial^2 u}{\partial y^2} = xy$ (ii) $1000K^4 \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ (iii) $\frac{\partial^2 u}{\partial x^2} + y \frac{\partial^2 u}{\partial y^2} = 0$ (iv) $\frac{\partial u}{\partial t} + t \frac{\partial^2 u}{\partial t \partial x} = \frac{\partial^2 u}{\partial x^2}$ **[10 marks]**

(b) Let $f\left(x, t, u, u_1, u_2, u_3 ...., u_m\right) = 0$ be an m^{th} order partial differential equation and $F(u_{ij}) = 0$ describe a finite difference scheme to it.

Describe the conditions under which the numerical scheme will converge to the unique the solution of the given partial differential equation [10 marks]

Question2 [20marks]

Given the wave equation

$$\frac{\partial^2 u}{\partial t^2} = 4 \frac{\partial^2 u}{\partial x^2}, \qquad 0 < x < 1, \quad t > 0$$

subject to boundary conditions $u(0,t) = u(1,t) = 0$ for $t > 0$

and initial conditions $u(x,0) = \sin(®x)$, $u_t(x,0) = 0$, $0 < x < 1$, $t = 0$. $h = \Delta x$, $t = 0$.

(a) Construct the explicit finite difference scheme to it. [6 marks]

(b) Express the resulting finite difference equation to the initial boundary value problem in a molecular form. [2 marks]

(c) State the stability of the explicit finite difference scheme in part (i) above. [2 marks]

(d) Compute the approximations $U_{i,j}$; $j = 1$(first − time level) , $i = 0,1,2,3,4,5.$ to the exact solutions $u(x_i, t_j)$ using $h = \Delta x = 0.2$, $k = \Delta t = 0.05$ $h = \Delta x$. On the same table display values of $j, i, U_{i,j}$, $u(x_i, t_j)$, $\left|u(x_i, t_j) - U_{i,j}\right|$ where $u(x,t) = \sin(®x)\cos(2®t)$. Comment on the accuracy of the results obtained. [10 marks]

Question3 [20marks]

On the square $D = \{(x, y) : 0 \le x \le l, 0 \le y \le l\}$ consider the Dirichlet problem for the Poisson's equation,

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x, y) \ \ in\ D$$

$$u = g(x, y) \ on\ S$$

(a) Use finite difference method with equal mesh spacing $h = \Delta x = \Delta y = l/4$, defined on D to discretize the Dirichlet problem, assuming . $g(x, y) = 0$ on S . **[15 marks]**

(b)Show that difference scheme takes the form

$A\underline{U} = \underline{B}$: $A_{9\times 9}$ real , symmetric matrix, $U = U_{ij}$, $B = B_{ij}$; $i, j = 1,2,3,4.$

Deduce that the numerically computed solution $\underline{U}$ is unique **[5 marks]**

Question4 [20marks]

(a) Construct an explicit finite-difference scheme as applied to the heat -diffusion equation

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}, \qquad 0\le x\le 1, \qquad 0<t<0.5$$

subject to $u(0,t)=u(1,t)=0 \qquad 0<t<0.5$

and $u(x,0)=x(1-x) \quad 0\le x\le 1$. **[5 marks]**

(b)Obtain a molecular formula for problem (a) above applied to the solution grid over region $W=\{(x,t):\ 0\le x\le 1,\quad 0\le t\le 0.5\}$ with $r=k/h^2$; $h=\Delta x=0.2$; $k=\Delta t=0.01$.

State the stability of the molecular formula employed and hence compute the numerical solutions $U_{i\,j}$; for the three time levels $j=1,\ 2,3$. **[15 marks]**

Question5 [20marks]

Consider the two dimensional -diffusion equation

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}, \qquad 0\le x\le 4\ ,0\le y\le 4 \quad , \qquad t>0$$

subject to $u(0,y,t)=u(4,y,t)=u(x,0,t)=u(x,4,t)=1$ for $t>0$

and $u(x,y,0)=xy\ ,t=0$.

Construct an explicit finite difference schemes to it.

State the stability of the explicit finite difference schemes [14 marks]