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UNIVERSITY EXAMINATIONS 2012 / 2013

EXAMINATION FOR THE DEGREE OF MASTER OF SCIENCE IN APPLIED STATISTICS

SAS 809: EPIDEMIC MODELLING

Instructions:

- This paper consists of **FIVE** Questions
- Answer Any **THREE** Questions.
- Observe further instructions on the answer booklet.

QUESTION ONE [20 Marks]

(a) Explain clearly what is meant by the following;

i. Epidemic

ii. Endemic

iii. An Epidemic model

(6 Marks)

(b) Consider the SI model defined by the following equations;

$$\frac{dS(t)}{dt} = -\lambda(t)cS(t)$$

$$\frac{dI(t)}{dt} = \lambda(t)cS(t) - vI(t)$$

where $\lambda(t) = \frac{\beta I(t)}{N(t)}$, $N(t) = I(t) + S(t)$ and c and v are some constants. Show that if $S(t) \cong N(t)$, then

$$I(t) = I_0 e^{(\beta c - v)t}$$

and the time taken, if there is an epidemic, for the number of infected to double is

$$t_d = \frac{\ln 2}{(\beta c - v)}$$

(10 Marks)

(c) Using the results in question (b),or otherwise, determine the conditions under which an epidemic will arise. (4 Marks)

QUESTION TWO [20 Marks]

Consider the "general epidemic" model with susceptibles $S(t)$, infectives $I(t)$, and Removals $R(t)$. Assuming homogeneous mixing among these classes and with continuous time t, let

$$\begin{aligned} dS(t)dt &= -\beta S(t)I(t) \\ \frac{dI(t)}{dt} &= \beta S(t)I(t) - \alpha I(t) \\ \frac{dR(t)}{dt} &= \alpha I(t) \end{aligned}$$

with initial conditions $S(0), I(0), R(0) = (S_0, I_0, 0)$ and α and β are some constants. also let $N(t) = R(t) + S(t) + I(t)$.

(a) Explain this formulation with respect to the spread of diseases.

(6 Marks)

(b) Show that,

i. $\frac{dI(t)}{dt} = \beta I(t)(S - \rho)$, $\rho = \frac{\alpha}{\beta}$. (4 Marks)

ii. $S(t) = S_0 e^{\frac{-R(t)}{\rho}}$. (4 Marks)

iii. For $N(t)$ such that $N(t) = \rho + \upsilon$ where $\upsilon \ll \rho$ with $N(t) \cong S_0$;

$$R(\infty) = 2\upsilon$$

$$S(\infty) = \rho - \upsilon$$

Comment on your results. (6 Marks)

QUESTION THREE [20 Marks]

Consider the homogeneous continuous time simple SIR epidemic model with the variables $S(t)$, $I(t)$ and $R(t)$ having the usual meaning;

(a) i. Give the initial and the boundary conditions. Hence or otherwise write down the model equations. (6 Marks)

ii. Show that

$$\phi(t) = \frac{\phi_0}{\phi_0 + (1-\phi_0)e^{-\beta N t}}$$

where $\phi(t)$ denotes the disease prevalence rate at time t with $\phi_0 = \phi(0)$. β and N are some constants. (6 Marks)

(b) Let $T_1 = \inf\{t : I(t) > N-1\}$. Show that

$$T_1 = \frac{1}{\beta N}\ln\left(\frac{(N-1)(N-I_0)}{I_0}\right)$$

where I_0 is some constant. (4 Marks)

(c) Let t^* be the time when the epidemic curve assumes the maximum value. Find the maximum value of the curve. (4 Marks)

QUESTION FOUR [20 Marks]

(a) Differentiate between the fixed effects and the random effects chain Binomial Models applicable in epidemic modeling. (10 marks)

(b) Consider the epidemic Chain Binomial model.

Let $i_t : t = 0, 1, \ldots r, (i_{r+1} = 0)$, denote the number of infectives at time t since the onset of the epidemic, and $S(t) : t = 0, 1, \ldots r, (S_r = S_{r+1})$ be the number of corresponding susceptibles at time t.

Let q_i be the probability that a susceptible escapes infection when exposed to i infectives.

Determine Prob [epidemic chain is $i_0 \rightarrow i_1 \rightarrow \ldots i_r$] and deduce the expression for the probability assuming;

i. Reed Frost model

ii. Greenwood Model

(10 Marks)

QUESTION FIVE [20 Marks]

(a) Briefly explain the main features of each of the following classes of epidemic models;

i. Empirical models

ii. Deterministic models

iii. Stochastic Models

(10 Marks)

(b) Consider a household of size 7 with 2 zero generation cases;

i. List all the possible epidemic chains. (5 Marks)

ii. Determine the probability that the size of the epidemic chain is five.

(5 Marks)