KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2016/2017

FIRST SEMESTER EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE

SMA 335 : ORDINARY DIFFERENTIAL EQUATIONS I

DATE: Tuesday 22nd November 2016

TIME: 11.00 A.M. – 1.00 P.M.

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INSTRUCTIONS

ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS

Question one (30 marks)

a) Use variation of parameters method solve the differential equation $\frac{d^2y}{dx^2} + y = \sec x$ (5marks)

b) Solve the differential equation $\frac{dy}{dx} = \frac{y}{x} + x \sin \frac{y}{x}$ (5marks)

c) Obtain the general solution for the Bernoulli equation $x^2 dy + y(x + y)dx = 0$ (5marks)

d) The rate at which the ice melts is proportional to the amount of ice at the instant. Find the amount of ice left after 2 hours if half the quantity melts in 30 minutes. (5marks)

e) Show that the functions $1, x, x^2$ are linearly independent. Hence form the differential equation out of them. (5marks)

f) Show that $\frac{1}{y^3}$ is the integrating factor of the differential equation $(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0$. Hence solve (5marks)

Question Two (20 marks)

a) Obtain a general solution for the equation $(2x + y + 1)dx + (4x + 2y - 1)dy = 0$ (5marks)
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b) Find the solution of the homogenous equation $x^3\frac{dy}{dx} = y^3 + y^2\sqrt{y^2 - x^2}$, satisfying $y(4) = 5$. (6marks)

c) A string with 3kg mass is held stretched 0.6m beyond its natural length by a force of 20 Newton. If the string begins at its equilibrium position but a push gives it an initial velocity of 1.2m/s, find the position of the mass after t seconds. (5marks)

d) Find the solution of the second order initial value problem $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$, satisfying $y(0) = 2, \frac{dy}{dx}|_{(0)} = 5$. (4marks)

Question Three (20 marks)

a) Show that the differential equation $\frac{dy}{dx} = \frac{2+ye^{xy}}{2y-xe^{xy}}$ is exact and hence solve. (6marks)

b) Solve the initial value problem $xy' + 2y = x^3,\ y(1) = \frac{6}{5}$ (5marks)

c) A manufacturing company has found that the cost C of operating and maintaining the equipment is related to the length m of intervals between overhauls by the equation $m^2\frac{dC}{dm} + 2mC = 2,\ C = 4$ when m=2. Find the relationship between C and m. (5marks)

d) Obtain a particular solution of $\frac{dy}{dx} + 2xy = -xy^4$ satisfying y=1 when x=0. (4marks)

Question Four (20marks)

a) By eliminating the arbitrary constants, find the differential equation associated to:
   i) $y = e^{-4x}(c_1\cos 5x + c_2\sin 5x)$ (3marks)
   ii) $(x-a)^2 + (y-b)^2 = 49$. (5marks)

b) Use the method of undetermined coefficients to solve $y'' + y' - 6y = (28x - 24)e^{4x}$ (6marks)

c) Solve the differential equation $y'' - 2y' + y = x^{-2}e^x$ using the method of variation of parameters. (6marks)

Question Five (20 marks)

a) Find the order and degree of the differential equation $\left(\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2}\right)^{\frac{5}{2}} = \left(2x + y\frac{d^2y}{dx^2}\right)^4$. Justify whether this equation is linear or non linear. (6marks)

b) Determine the general series solution of the equation $(x^2+1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - 4y = 0$ (8marks)

c) i) Explain what is meant by the Wronskian of terms u_1, u_2 and u_3. Why is the Wronskian important? (3marks)

d) use the Wronskian to show if the solutions of a differential equation are e^{3x} and xe^{3x} then they are linearly independent. Give the general solution of the differential equation. (3marks)