KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2016/2017

FIRST SEMESTER EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE, BACHELOR OF EDUCATION, BACHELOR OF ARTS

SMA 330 : NUMERICAL ANALYSIS I

DATE: Monday 28th November 2016

TIME: 8.00 a.m. – 10.00 a.m.

INSTRUCTIONS

ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS

Question One (30marks)

- a) Define the following terms as used in computation.
 - i) Relative error
 - ii) Reformulation
 - iii) Interpolation (3 marks)
- b) i) Use three digit rounding arithmetic to perform the calculation below: $\left(\frac{13}{14} - \frac{6}{14}\right) \div (2e - 5.4)$
 - ii) Compute the percentage error with the exact value determined to five digit rounding. (5marks)
- c) Two qualities are found to be related as below

x _i	1.00	1.05	1.10	1.15	1.20
$f(x_i)$	1.0000	1.0240	1.0488	1.0724	1.0956

Assuming y = f(x) is continuous find

 $\int_{1.0}^{1.2} f(x) dx$, numerically using both trapezoidal and Simpsons rule.

(4 marks)

- d) Write an expression for μ and δ in terms of the differential operator E. Hence show that $\mu \delta = \frac{1}{2} (\Delta + \nabla).$ (3 marks)
- e) Find the value of $\sin 52^\circ$ from the given data by using Newton's forward interpolation.

- f) i) Show that $e^x 2x 7 = 0$ has a root between 2 and 3
 - ii) Show that $x_{n+1} = 1_n(2x_n + 7)$ is convergent fixed point iteration formula hence use the formula to find the root. Let $x_0 = 2.5$ (6marks)
- g) construct a difference table for the data below and use it to determine the collocation polynomial for the data.

x	0	1	2	3	4	5	6
f(x)	2	3	10	29	66	127	218
					(4)	marks)	

Question Two (20marks)

a) i) Given the polynomial $f(x) = x^4 - 2x^3 + 3x^2 - x + 1$ for x = -3(1)4, use synthetic division to find the missing values of f(x) in the table below.

x	-3	-2	-1	0	1	2	3	4
f(x)	166	-	8	1	2	11	-	173

(4 marks)

ii) Form a difference table for the data in (i) above and use it to compute f(2.5) (5 marks)

b) The data below has one incorrect entry of f(x). Locate and correct the wrong value.

x	0	1	2	3	4	5	6	7
f(x)	10.60	10.99	11.52	12.25	13.42	14.55	16.24	18.37

(6marks)

c) Show that the equation $x^3 + 4x^2 - 10 = 0$ has a root in the interval [1, 2]. Perform five iterations of the bisection method to obtain a positive root to the equation correct to 4 d.p. (5 marks)

Question Three (20marks)

- a) Prove the relation $f'(x_0) = \frac{1}{h}\Delta f_0 = \frac{f_1 f_0}{h}$ (3 marks)
- b) The table below gives the values of f(x)

x	0.0	0.1	0.2	0.3	0.4	0.5	
f(x)	1 57080	1 52076	1 48004	1 1/1522	1 20020	1 25064	
f(x)	1.37080	1.33070	1.48904	1.44333	1.39939	1.55004	
e the table to find $f'(0.3)$ and $f''(0.3)$. (5 marks)							

Use the table to find f'(0.3) and f''(0.3).

- c) Find the approximate value of the integral $\int_0^1 \frac{1}{1+x} dx$ using composite trapezoidal rule with 2,3,5 and 9 nodes and Romberg integration. Compare the results with the exact value. (8 marks)
- d) Evaluate the integral $\int_{-1}^{1} e^{-x^2} \cos x \, dx$ using Gauss-Legendre three point formula. (4marks)

Question Four (20marks)

a) The table below gives the value of x_i and the corresponding values of $f(x_i)$ for a polynomial p(x).

x _i	0	2	3	4
$f(x_i)$	7	11	28	63

- i) Use Langrange's interpolation process to obtain the polynomial p(x) that fits the data. (5 marks)
- Construct the divided difference table for the data and use it to find the Newton's ii) interpolating polynomial. (5 marks)
- Show that the polynomials in (i) and (ii) are identical although their forms may iii) be different. (2marks)

- b) i) Show that the equation $2x^3 5x^2 3$ is solvable by the Newton Raphson iterative formula $x_{n+1} = \frac{4(x_n)^3 - 5(x_n) + 3}{6(x_n)^2 - 10(x_n)}$ (3 marks)
 - ii) Show that the equation above has a root in the interval [2, 3] and find this root correct to 4 d.p. taking $x_0 = 2.5$ (5 marks)

Question Five (20marks)

- a) i) Distinguish between the terms inherent error and accumulated error as used in computation. (2marks)
 ii) Obtain the range of values within which the exact value of
 - $\frac{1.265 (10.21 7.57)}{47}$ lies if all the numeral quantities are rounded off. (4marks)
 - A rectangular parallel pipe has sides 3cm, 4cm, and 5cm measured to the nearest cm. What are the best upper and lower bounds for the volume of the parallel pipe? (4marks)
- b) Evaluate the integral $I = \int_0^6 \frac{dx}{1+x^2}$ by using
 - i) Trapezoidal rule
 - ii) Simpson's one third rule with h=1

- (6marks)
- c) Using direct integration, evaluate the exact integral in (b) above correct to 5 decimal places and hence find the percentage error in each of the approximated values in (b). (4 marks)