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UNIVERSITY EXAMINATIONS 2012 / 2013

EXAMINATION FOR THE DEGREE OF MASTER OF SCIENCE IN APPLIED STATISTICS

SAS 812: NON-PARAMETRIC TESTS

Instructions:

- This paper consists of **FIVE** Questions
- Answer Any **THREE** Questions.
- Observe further instructions on the answer booklet.

QUESTION ONE [20 Marks]

(a) Explain the meaning of the following terms;

i. Non-parametric Test

ii. p-value

iii. Order Statistic

(6 Marks)

(b) Find the smallest value of n for which $pr(y_1 < \eta_{0.5} < y_n) \geq 0.99$, where $y_1 < y_2 < \ldots y_n$ are the order statistics of a random sample of size n from a distribution of the continuous type and $\eta_{0.5}$ is the 50-th percentile of the distribution. (6 Marks)

(c) The following data (in tons), are the amounts of sulfur oxides emmitted by a large industrial plant in 40 days;

17	15	20	29	19	18	22	25	27	9
24	20	17	6	24	14	15	23	24	26
19	23	28	19	16	22	24	17	20	13
19	10	23	18	31	13	20	17	24	14

Test the null hypothesis that $\mu = 21.5$ against the alternative that $\mu < 21.5$ at 0.01 level of significance. State any assumptions you may have made.

(8 Marks)

QUESTION TWO [20 Marks]

(a) Briefly explain the concept of runs. Find an expression for the probability that n_1 letters of one kind and n_2 letters of another kind will form U runs when each of the $\binom{n_1+n_2}{n_1}$ possible arrangements of these letters is regarded as equally likely, and U is even. (5 Marks)

(b) Find the probability that $n_1 = 6$ letters of one kind, and $n_2 = 5$ letters of another kind will form at least 8 runs. (5 Marks)

(c) The following are speeds (in Km per hour) at which every fifth passenger car was timed at a certain check point. Data is read column-wise.;

46	58	60	56
70	66	48	54
62	41	39	52
45	62	53	69
65	65	65	67
76	52	52	59
59	67	51	46
61	40	43	42
77	67	63	63
72	57	59	42
56	47	62	67
70	63	66	69

Test the null hypothesis of randomness at $\alpha = 0.01$ (10 Marks)

QUESTION THREE [20 Marks]

(a) Prove the Probability Integral Transform by finding the m.g.f of $Y = F(x)$, where x has the continuous distribution $F(x)$. (5 Marks)

(b) If X is a continuous random variable with p.d.f $f(x) = 2(1 - x), \quad 0 < x < 1$, find the transformation $Y = g(x)$ such that the random variable has the uniform distribution over (0,2).

(5 Marks)

(c) Find the mean and variance of the median of a random sample of size n from the uniform population over (0,1);

i. when n is odd.

ii. when n is even.

(5 Marks)

(d) Find the expected value of the largest order statistic in a random sample of size 3 from $Z \sim N(0, 1)$.

(5 Marks)

QUESTION FOUR [20 Marks]

(a) Compare the one-sample sign test, and the wilcoxon signed rank test.

(3 Marks)

(b) Consider two independent random variables X and Y with distributions F(X) and G(Y) respectively.

Let $(x_i, y_i), i = 1, 2 \ldots n$ be pairs of independent observations on (X, Y), then define $D = X - Y$, and $D_i = x_i - y_i$.

If θ is the median of the distribution of D, and assuming that the distribution of D is symmetrical about θ, show that

$$E(T^+|\theta = 0) = \frac{n(n+1)}{4}$$

and

$$Var(T^+|\theta = 0) = \frac{n(n+1)(2n+1)}{24}$$

where T^+ is the sum of the ranks of the positive differences $D_i > 0$. State any assumptions that you may have made. (7 Marks)

(c) The Human Rights Council (HRC) is interested in the relationship between different cities' average per capita income and their murder rate. A random sample of 8 cities produced the following values of average per capita income (in 1000USD) and murder rate per 100,000 inhabitants;

Income	22.56	23.07	23.48	25.33	26.13	27.21	30.75	31.81
Murder Rate	13.01	13.32	14.49	12.72	12.05	12.64	8.97	10.09

i. Calculate the Spearman's rank correlation coefficient r_s, between average per capita income and murder rate.

(6 Marks)

ii. Test whether significant negative Spearman's rank correlation exists.

(4 Marks)

QUESTION FIVE [20 Marks]

(a) A manufacturer of a certain type of disk drives claims that its disk has an average lifetime of 2.3 years under conditions of continuous use. The publishers of a computer magazine believe this claim is exaggerated. A random sample of nine of disk drives produced the following lifetimes (in years) under continuous use;

1.93 2.02 2.01 2.12 2.14 1.89 1.79 2.06 2.00

Using the sign test, determine whether the mean lifetime of this type of disk drive is actually less than 2.3 years. (10 Marks)

(b) The following frequency distribution gives the number of cars arriving at a certain junction of a road in 60 seconds intervals.

No. of cars	0	1	2	3	4	5	6	7	8	9	10
Observed Frequency	8	23	39	53	36	30	15	7	5	3	1

Can you conclude that the Poisson distribution provides a suitable distribution?

Take $\alpha = 0.025$. (10 Marks)