

MOI UNIVERSITY

SCHOOL OF BIOLOGICAL & PHYSICAL SCIENCES

DEPT. OF STATISTICS & COMPUTER SCIENCE

UNIVERSITY EXAMINATIONS

2014/2015 ACADEMIC YEAR, NOV/DEC 2014

**THIRD YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF
BACHELOR OF SCIENCE (ACTUARIAL SCIENCE)**

COURSE CODE: ACS 302

COURSE TITLE: ACTUARIAL MATHEMATICS II

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INSTRUCTIONS TO CANDIDATES

Answer **ALL** questions from section A and any **THREE** from section B

Duration of the examination: 3 hours

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SECTION A (31 Marks) :(Two questions, one carrying 15 Marks and the other 16 Marks)

Answer Both Questions

QUESTION 1

(16 Marks)

a).A certain life office sells assurance policies with term 3 years to lives aged 70. For each policy, the profit vector is estimated to be $(-50, 30, 30)$. Given that the mortality of the policy holder is expected to follow a 1967-70 ultimate, calculate:

- i. The profit signature per policy sold.
- ii. The net present value of the profit to the office on the basis of a risk discount rate of 8% per annum. (7 marks)

b).Assume that l_x individuals aged x each put the same amount of money in a fund. This fund is left to accumulate for n years at rate i , and is then divided up among the l_{x+n} survivors. Give an expression for the payment to be made per survivor. (2 marks)

c) Briefly explain the meaning of the following terms as applied to the theory of

Sickness Functions:

- i. The Deferred Period (1 Marks)
- ii. The waiting Period (2 Marks)
- iii. The off-period (2 Marks)
- d) Briefly mention the meaning of a *Joint Life Status* (2 Marks)

QUESTION 2

15 Marks

- a). Given that ${}_n P_x = 0.3$, ${}_n P_y = 0.6$, ${}_n P_z = 0.4$, find the probability that, of the lives (x), (y) and (z),
- i). none will survive n years (1 Mark)
 - ii). exactly one will survive n years (2 Marks)
 - iii). At least one will survive n years (2 Marks)
- b). Express in terms of P_x , P_y and P_z the probabilities that, of three lives (x), (y) and (z),
- i). All three will survive one year 1 Mark
 - ii). At least one will survive one year 1 Mark
 - iii) Exactly two will survive one year 1 Mark
- c). A joint life annuity of 1 per annum is payable continuously to lives currently aged (x) and (y) while both lives are alive. The present value of the annuity payments is expressed as a random variable, in terms of the joint future lifetime of the two lives. Derive and simplify as possible expressions for the expected present value and variance of the present value of the annuity.
- (5 Marks)

SECTION B

(39 Marks)

(ANSWER ANY THREE OUT OF THE FIVE QUESTIONS)

QUESTION 3

(13 Marks)

a). In the context of profit-testing, explain the difference between the “*profit vector*” and the “*profit signature*”. (2 Marks)

b). A life office is proposing to issue 3-year sickness benefit policies to lives aged 30. The benefits are \$ 50 per week during sickness within the next three years. There is no waiting period and the off period is as in the tables provided.

a). i). Find the single premium on each of the following bases:

Mortality: English life Table No.12 –Males

Interest: i) 4% p.a

ii) 5 % p.a,

Sickness: Manchester Unity 1893-97 (AHJ)

Expenses: None (4 Marks)

ii). Calculate the value of benefits from first principles (4 Marks)

c). An illness-death model has three states:

1=healthy 2=sick 3=dead

Draw and label a diagram showing the three states and the transition intensities between them.

3 Marks

QUESTION 4

(13 Marks)

a). A friendly society issued a policy providing the following benefits to a man aged exactly (25) at entry.

i). On death at any time before age 60, the sum of \$ 4,000 payable immediately.

ii). On survival to age 60, an annuity of \$8 per week payable weekly in advance as long as he survives;

iii). On sickness an income benefit to be payable during sickness of \$ 32 per week for the first 6 months reducing to \$ 16 per week for the next 18 months and to \$8 per week thereafter sickness benefit is not payable after age 60. There is no waiting period. Premiums are payable monthly in advance for at most 35 years and are not waived during periods of sickness. The society uses the following basis to calculate premiums. Find the monthly premium.

Mortality: English life Table No-12- males

Sickness: Manchester Unity Sickness.

Experience 1893-97, occupation group AHJ

Interest: 4% pa

Expenses: None

Find the monthly premium. (7 Marks)

b) i. State the age ranges over which the Gompertz' law is an appropriate model for human mortality. (1 Mark)

ii) Show that, under Gompertz' law, the probability of survival from age x to age $x+n$ is equal to $[\exp(-B/\log c)]^{cx(ct-1)}$ (5 Marks)

QUESTION 5 (13 MARKS)

a). Extra Risks may be treated as an additional Mode of Decrement. Briefly mention an example of this situation. 2 Marks

b). A certain life office's premium basis for policies accepted at normal rates is:

A 1967-70 select 4% interest Expenses are ignored

A proposer, aged 45, for temporary assurance ceasing at age 65 is subject to an extra occupational hazard which is considered to be equivalent to an addition of 0.009569 to the force of mortality at all ages. The sum assured, which is payable immediately on death, is £ 10000.

a). Calculate the level annual premium, payable throughout the term of the policy. 8 Marks

b). The proposer requests that, in the event of death occurring as a result of the special occupational hazard, the sum assured should be doubled, and offers to pay an additional single premium at the outset for this extra cover. Calculate this single premium. 3 Marks

QUESTION 6

13 Marks

a). Substantiate between: *Defined-benefit schemes* and *Defined-contribution schemes* 2 Marks

b) Consider a person now aged exactly 25 whose annual salary rate is currently \$9192

Estimate:

i). His annual salary rate at exactly age 53 1 Mark

ii). His earnings between exact ages 64 & 65 2 Marks

iii). The average amount earned by him each year between exact ages 60 and 65 2 Marks

iv). The total amount earned over his last year of life 2 Marks

(Assume that salaries are revised continuously and assume 4% per annum interest)

C). The probability that at least one of three lives aged 60 will survive to age 65 is eight times the probability that exactly one will survive to age 65. Assuming that the three lives are independent and subject to the same table of mortality, find the probability that exactly one life will survive to age 65 4 Marks

QUESTION 7

13 Marks

a). The members of a large company's manual workforce are subject to three modes of decrement, death, withdrawal and promotion to supervisor. It is known that these workers' independent rates of mortality are those of English life Table No. 12 Males, the independent withdrawal rate is 0.04 at each age, and their independent promotion rate is 0.02 at age 50 and 0.03 at age 51.

i). Draw up a service table for manual workers from age 50 to age 51 with a radix of 100,000 at age 50, including the value of $(al)_{52}$ 7 Marks

ii). Calculate the Probability that a life aged 50 will gain promotion within 2 years. 3 Marks

b). Briefly explain the meaning of *Dependent rates of exit* 3 Marks