**MAIN CAMPUS ACS 304 MAIN EXAMINATION**

**MOI UNIVERSITY**

**SCHOOL OF BIOLOGICAL AND PHYSICAL SCIENCES**

**DEPT. OF PHYSICS, MATHEMATICS, STATISTICS & COMPUTER SCIENCE**

**UNIVERSITY EXAMINATIONS**

**2013/2014 ACADEMIC YEAR, AUGUST/SEPTEMBER 2014 SERIES**

**THIRD YEAR FIRST SEMESTER EXAMINATION FOR THE DEGREE OF**

**BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE**

**COURSE CODE: ACS 304**

**COURSE TITLE: FINANCIAL MATHEMATICS II**

**INSTRUCTION TO CANDIDATES**

Answer ALL questions from section A and any THREE from section B.

Duration of the examination: 3 hours

***YOU ARE ALLOWED TO HAVE AVAILABLE ACTUARIAL TABLES AND A VALID ELECTRONIC CALCULATOR***

**Section A---- COMPULSORY-- (31 Marks)**

ANSWER **BOTH** QUESTIONS IN THIS SECTION

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**QUESTION 1-Compulsory (16 Marks)**

1. i). What do you understand by a ***Stochastic Model?*  1 Mark**

ii). A stochastic Interest Rate Model assumes that the interest rates in different years are independent and identically distributed normal random variables with mean 8% and standard deviation 2%. Find the standard deviation of the accumulated value at time 2, of an investment of KShs 12000 **6 Marks**

1. A stochastic interest rate model assumes that the annual growth factors for each future year are independently distributed log-normal random variables with parameters μ=0.05 and Ϭ2=0.01. Calculate the mean and standard deviation of the annual rate of return.

**5 Marks**

1. One of the concepts of Portfolio risk introduced by Markowiz is that:

Var (Rp) = (σ2/n) [1+ (n-1) ρ].

State the meaning of the notations: N, p, Ϭ and RP.  (**4 Marks)**

**Question 2 [15 Marks**

a).i). Explain what is meant by the term structure of interest rates. **(2 Marks)**

ii). If **F t, r** is the force of interest corresponding to the annualized forward rate of interest **f r,t** and **Pt** is the price of a unit zero-coupon bond of term t, show that: f r,t  = eFt,r – 1

and that Ft,r =1/r log{Pt/Pt+r}  **(6 Marks)**

b) i) Briefly substantiate between Macaulay Duration and Modified duration. Hence derive the formula relating the two types of duration. **(7 Marks)**

**SECTION B- 39 Marks-(ANSWER ANY THREE OUT OF THE FIVE QUESTIONS FROM THIS SECTION). EACH QUESTION CARRIES 13 MARKS**

**Question 3 [13 Marks**

a). The 1, 2,3,4 and 7-year spot rates are4%, 3.75%, 3.6% and 3.9% per annum respectively.

The 3-year forward rate from time 3 is 3.8% per annum and the 3-year forward rate from time 2 is 3.7% per annum. Calculate:-

1. the 2-year forward rate from time 5 **(4 Marks)**
2. the present value of payments of 5 at the end of each of the 7 years **(3 Marks)**
3. the accumulated value at time 6 of a payment of 100 at time 1 **(2 Marks)**

b). The following forward rates are expressed as rates compounded twice per year:-

**fo(2)=6%, f1(2)= 5%, f2(2)= 7%**

Find the 1-year, 2-year and 3-year spot rates, expressed as rates compounded four times per year.

**(4 Marks)**

**Question 4 [13 Marks]**

1. CAPITAL brokers-Eldoret Ltd approach you for advice on investment of their wealth. Two mutually exclusive investment opportunities A and B are available. On analysis you discover that each of their two investment opportunities has an expected rate of return of 19.5% and a variance of 420. Advise the investor on the way forward. **(3 Marks)**
2. Explain any 2 assumptions of the Capital Asset Pricing Model  **(4 Marks)**
3. Explain the 3 weaknesses of the Capital Assets Pricing Model (CAPM). **(6 Marks)**

**Question 5 [13 Marks]**

1. An insurance company is modeling its investment performance over the next 10 years by assuming that the yields obtained during the first 5 years will have a constant value of 4%, 5% or 6% per annum, each with equal probability, and that the yield during the second 5 years will be 1% lower than the yield during the first 5 years. Find the

expected accumulated value at the end of 10 years of a single investment of KShs 100,000 at time 0. (**7 Marks)**

1. The n-year spot rate Sn is given by the formula: Sn=0.050-(n/500); for n=1, 2 and 3. Calculate the implied one-forward rates at times t=1 and t=2 **(6 Marks)**

**Question 6 [13 Marks]**

a). i) According to the portfolio theory, **“*Investors are Risk-Averse”*****(2 Marks)**

ii)Briefly explain the two assumptions of the Portfolio Theory. **(4 Marks)**

b). Explain the features and mode of operation of the **A**rbitrage **P**ricing **M**odel **(4Marks)**

c). Suppose the possible outcomes of two Assets X and Y under different economic states A, B, C, D E are as follows:

|  |  |  |  |
| --- | --- | --- | --- |
| **State of ECOMOMY** | **PROBABILITY** | **Return (%)** | |
| **X** | **Y** |
| A | 0.10 | -8 | 14 |
| B | 0.20 | 10 | -4 |
| C | 0.40 | 8 | 6 |
| D | 0.20 | 5 | 15 |
| E | 0.10 | -4 | 20 |

1. Give an expression or formula for computing the expected rate of return of an individual asset.  **(2 Marks)**
2. Hence compute the expected rate of return of asset X. **(1 Mark)**

**Question 7 [13 Marks]**

a). Suppose an investor decides to invest 50% of his wealth in asset X and 50% in asset Y. Compute this investors’ expected rate of return on a portfolio consisting of both X and Y.

**(2 Marks)**

b) By adopting a particular investment strategy, a company expects that on average the annual yield on its funds will be 8%. However, the investment policy is one of comparatively high risk and the standard deviation of the annual yield is expected to be 7%. Assuming the yields in different years to be independently distributed:

(I). Find the expected value and standard deviation of the accumulated amount after 15 years of:-

i) A single premium of KShs. 1,000  **(4 Marks)**

ii) A series of 15 annual premiums each of KShs. 1,000  **(3 Marks)**

Assuming further that each year 1+I has a log-normal distribution,

(II). Calculate the probability that a single premium accumulation will be less than 60% of its expected value.  **(4 Marks)**