BONDO UNIVERSITY COLLEGE

SCHOOL OF BIOLOGICAL AND PHYSICAL SCIENCES

DEPARTMENT OF PHYSICS

UNIVERSITY EXAMINATION 2012

THIRD YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION WITH I.T

SPH 303: QUANTUM MECHANICS I

(REGULAR PROGRAM)

INSTRUCTION

1. Answer question **ONE** in **SECTION A** and any other **TWO** questions from **SECTION B**.
2. Question **ONE** in **SECTION A** carries **30 MARKS** and the questions in **SECTION B** carry **20 MARKS** each.

Apply appropriately

Plank's constant	$h = 6.626 \times 10^{-34}\ Js$
Charge on electron	$e = 1.602 \times 10^{-19}\ C$
Mass of electron	$m_e = 9.11 \times 10^{-31}\ kg$

SECTION A. (30 MARKS) **ANSWER ALL QUESTION IN THIS SECTION**

QUESTION ONE

(a) State the fundamental characteristics of the wave equation satisfied by the wave function $\psi(r,t)$. (3 marks)

(b) For an electron in a one-dimensional infinite potential well of width $1\overset{0}{A}$, calculate the frequency of the photon corresponding to a transition between the two lowest energy levels. (3 marks)

(c) For a particle moving in a straight line with energy $E = \frac{1}{2}mv^2$, show that $\Delta E \Delta t \geq \frac{h}{4\pi}$. (3 marks)

(d) Normalize the wave function $\Psi(x) = Ax\exp\left(-ax^2\right)$, where A and a are constants over the domain $0 \leq x \leq +\infty$. (3 marks)

Hint $\int_0^\infty x^2 \exp\left(-ax^2\right)dx = \frac{\sqrt{\pi}}{4a^{3/2}}$

(e) Show that two commuting operators A and B have a common set of eigenfunctions. (3 marks)

(f) Consider a stream of particles of mass m approaching a square potential barrier from left. The potential $V(x)$ is defined by

$$V(x) = \begin{cases} 0 & x < 0 \\ V_0 & 0 < x < l \\ 0 & l < x \end{cases}$$

Explain the quantum mechanical tunneling effect. (3 marks)

(g) Evaluate $\left[\frac{d^2}{dx^2}, ax^2 + bx + c\right]$ (3 marks)

(h) Consider a particle of mass m moving inside a square potential well with finite barrier of height V_0

$$V(x) = \begin{cases} V_0 & x > -a \\ 0 & -a < x < a \\ V_0 & x > a \end{cases}$$

Express that the stationary state Schrödinger equation for the different regions when $E < V_0$ (3 marks)

(i) Describe Bohr correspondence principle. . (3 marks)

(j) Show that the function $\exp(i\mathbf{k}.\mathbf{r})$ is an eigenfunction of the operator $-\hbar^2\nabla^2$ and find its eigenvalue. (3 marks)

SECTION B. ANSWER ANY TWO QUESTIONS.

QUESTION TWO

(a). The general equation of a wave propagating in the $x-$direction is

$$\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dx^2} \qquad (1)$$

where v is the velocity of propagation. For a restricted wave the solution to (1) is

$$y = Ae^{-i\omega\left(t - x/v\right)} \qquad (2)$$

where ω is the angular frequency.

(i) Apply equations (1) and (2) for wave associated with a particle to develop Schrödinger time dependent equation. (11 marks)

(ii) Using the results in 2 (a) (i) obtain the steady state form of the Schrödinger equation. (5 marks)

(b) A particle has a wave function $\psi(x) = Ce^{-x/a}$ for $x \geq 0$ and $\psi(x) = 0$ for $x \leq 0$. Determine the normalization constant and the average value of the position x.

Hint $\int_0^\infty x^n e^{-bx} dx = \frac{n!}{b^{n+1}}$ (4 marks)

QUESTION THREE

(a) Consider a current of particles of mass m confined in an infinite one-dimensional potential well of width a;

$$V(x) = \begin{cases} 0 & -a/2 \leq x \leq a/2 \\ \infty & everywhere \end{cases}$$

(i) Find the eigenstates of the Hamiltonian (i.e. the stationary states) and the corresponding eigenenergies. (12 marks)

Hint $\int_{-a/2}^{a/2} \sin^2\left(\frac{n\pi}{a}x\right)dx = \int_{-a/2}^{a/2} \cos^2\left(\frac{n\pi}{a}x\right)dx = \frac{a}{2}$

(ii) Draw a graphical representation of the eigenstate obtained in 3 (a) (i) for $n = 1, 2$ *and* 3 in order to obtain quantitative properties of the solution. (3 marks)

(b) Evaluate the commutator $\left[\hat{x}, \hat{p}_x^2\right]$. (5 marks)

QUESTION FOUR

(a) Consider a square potential barrier defined by

$$V(x) = \begin{cases} 0 & x < 0 \\ V_0 & 0 < x < l \\ 0 & l < x \end{cases}$$

A stream of particles of mass m and energy $E > V_0$ approaches the square barrier from $x = -\infty$.

(i) Find the stationary states of the system. (8 marks)

(ii) Use the matching condition to express the reflected amplitude in terms of the incident amplitude. (8 marks)

(b). Prove that for the operators $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$:

$$[[\mathbf{A},\mathbf{B}],\mathbf{C}] + [[\mathbf{B},\mathbf{C}],\mathbf{A}] + [[\mathbf{C},\mathbf{A}],\mathbf{B}] = 0$$

(4 marks)

QUESTION FIVE

(a) (i) Define the uncertainty (ΔA) in the measurement of a dynamical variable. State and explain the general uncertainty relation. (4 marks)

(ii) A particle is in a state $|\psi\rangle = \left(\frac{1}{\pi}\right)^{1/4} \exp\left(-\frac{x^2}{2}\right)$. Find Δx and Δp_x. Hence evaluate the uncertainty product $(\Delta x)(\Delta p_x)$. (10 marks)

Hint $\int_{-\infty}^{+\infty} x^n e^{-ax^2} dx = 0 \quad \text{If } n \text{ is odd}$

$$\int_{-\infty}^{+\infty} e^{-ax^2} dx = \left(\frac{\pi}{a}\right)^{1/2}$$

$$\int_{-\infty}^{+\infty} x^2 e^{-ax^2} dx = \frac{\sqrt{\pi}}{2a^{3/2}}$$

(b) The normalized wave function of a particle trapped in a one-dimensional box is

$$\psi_n(x) = \sqrt{\frac{2}{l}} \sin\left(\frac{n\pi}{l}\right)x$$

Determine whether the wave function is a momentum or total energy eigenfunction (6 marks)