BONDO UNIVERSITY COLLEGE

SCHOOL OF BIOLOGICAL AND PHYSICAL SCIENCES

DEPARTMENT OF PHYSICS

UNIVERSITY EXAMINATION 2012

THIRD YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION WITH I.T

SPH 303: QUANTUM MECHANICS I

(SCHOOL BASED PROGRAM)

INSTRUCTION

1. Answer question **ONE** in **SECTION A** and any other **TWO** questions from **SECTION B**.
2. Question **ONE** in **SECTION A** carries **30 MARKS** and the questions in **SECTION B** carry **20 MARKS** each.

Apply appropriately

Plank's constant	$h = 6.626 \times 10^{-34}\ Js$
Charge on electron	$e = 1.602 \times 10^{-19}\ C$
Mass of electron	$m_e = 9.11 \times 10^{-31}\ kg$

SECTION A. (30 MARKS)

ANSWER ALL QUESTION IN THIS SECTION

QUESTION ONE

(a) Outline the probability interpretation of the wave function. (3 marks)

(b) State the quantum mechanical analogue of the classical energy expression $E = p^2/2m + V(r)$ and explain its significance. (3 marks)

(c) The average lifetime of an excited atomic state is $10^{-9}\ s$. If the spectral line associated with the decay of this state is $6000\ \overset{0}{A}$, estimate the uncertainties in the wavelength of the line. (3 marks)

(d) Normalize the wave function $\Psi(x) = A\exp(-ax^2)$, where A and a are constants over the domain $-\infty \leq x \leq +\infty$. (3 marks)

Hint $\int_{-\infty}^{+\infty} \exp(-2ax^2)dx = \sqrt{\pi/2a}$

(e) A particle of mass $2.00 \times 10^{-26}\ kg$ is in a one-dimensional box of length $4.00\ nm$. Find the frequency of the photon emitted when the particle goes from $n = 3$ to $n = 2$ level. (3 marks)

(f) Explain the quantum mechanical tunneling effect. (3 marks)

(g) Find $\left[z^3, d/dz\right]$ (3 marks)

(h) Describe the transmission and reflection coefficients of a particles (or a stream of particles) moving through a potential step. (3 marks)

(i) Find the eigenfunction and eigenvalue of the operator d/dx . (3 marks)

(j) Prove that the eigenvalues of Hermitian operator are real. (3 marks)

SECTION B.

ANSWER ANY <u>TWO</u> QUESTIONS. EACH QUESTION CARRIES 20 MARKS.

QUESTION TWO

(a). Consider a particle subjected to a time-independent potential $V(r)$ Assume that a state of a particle is described by a wave function of the form $\Psi(r,t) = \phi(r)\chi(t)$.

(i) Show that if $\chi(t) = Ae^{-i\omega t}$ (A is a constant) then $\phi(r)$ must satisfy the equation

$$\frac{-\hbar^2}{2m}\nabla^2\phi(r) + V(r)\phi(r) = \hbar\omega\phi(r)$$

where m is the mass of the particle (5 marks)

(ii) Prove that the solution of the Schrödinger equation of part 2 (a) (i) leads to a time-independent density. (3 marks)

(b) (i) Calculate the time derivative of the probability density and show that continuity equation

$$\frac{d\rho(r,t)}{dt} + \nabla . j(r,t) = 0$$

is valid. (8 marks)

(ii) Find the probability current density corresponding to the wave function

$$\Psi(x,t) = Ae^{i\frac{p_x}{\hbar}x}e^{i\omega t}$$

(4 marks)

QUESTION THREE

Consider a current of particles of energy $E > V_0$ moving from $x = +\infty$ to the right toward a step potential

$$V(x) = \begin{cases} V_0 & x > 0 \\ 0 & x < 0 \end{cases}$$

(i) Write the stationary solution for the two regions. (6 marks)

(ii) Explain the fat that there is no current coming from $x = +\infty$ to the left. (4 marks)

(iii) Use the matching condition to express the reflected and transmitted waves in terms of the incident amplitude. (7 marks)

(iv) The reflection coefficient is defined as $R = \left|\frac{B}{A}\right|^2$, where A and B is the incident and reflected amplitudes respectively. Using the results in (iii) obtain the transmission coefficient. (3 marks)

QUESTION FOUR

(a). The normalized wave function of a particle trapped in a one-dimensional box is

$$\psi_n(x) = \sqrt{\frac{2}{l}} \ \sin\left(\frac{n\pi}{l}\right)x$$

(i) Determine whether the wave function is a momentum or kinetic energy eigenfunction (6 marks)

(ii) Calculate the expectation value $\langle \hat{x} \rangle$ of the position of the particle.

Hint $\int_0^l x \cos\left(\frac{2n\pi}{l}\right)x \, dx = 0$ (6marks)

(b). Prove that for the operators $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ the following identities are valid

(i) $[\mathbf{B}, \mathbf{A}] = -[\mathbf{A}, \mathbf{B}]$ (2 marks)

(ii) $[\mathbf{A} + \mathbf{B}, \mathbf{C}] = [\mathbf{A}, \mathbf{C}] + [\mathbf{B}, \mathbf{C}]$ (2 marks)

(c) Derive the fundamental quantum mechanical commutation bracket $[\hat{x}, \hat{p}_x] = i\hbar$.

(4 marks)

QUESTION FIVE

(a) Describe the postulates of quantum mechanics. (12 marks)

(b) Consider a particle of mass m moving inside a potential well with finite barrier of height V_0

$$V(x) = \begin{cases} V_0 & x > -a \\ 0 & -a < x < a \\ V_0 & x > a \end{cases}$$

Find the wave function of the particle for the different regions when $E < V_0$

(8 marks)