QUESTION ONE (30 MARKS)

a) If $\bar{a} = 2i + 3j + 4k$ and $\bar{b} = 5i - 6j + 7k$ evaluate (6 marks)

    i. $\bar{a} . \bar{b}$

    ii. $\bar{a} \times \bar{b}$

b) Calculate the angle between the two vectors $\bar{a} = 2i + 2j - k$ and $\bar{b} = 6i - 3j + 2k$ (5 marks)

c) Show that the vectors $\bar{a} = 3i - 2j + k$ and $\bar{b} = 2i + j - 4k$ are orthogonal.   (3 marks)

d) Solve for x and y using method of determinants   (5 marks)

$$x + 2y = -4$$
$$5x + 3y = 1$$

e) If $\underline{w} = 3t^2 i + \cos 2tj$, find   (6 marks)

    i) $\dfrac{d\underline{w}}{dt}$

    ii) $\left| \dfrac{d\underline{w}}{dt} \right|$

    iii) $\dfrac{d^2 \underline{w}}{dt^2}$

f) If $r = 3ti + t^2 j + (1 + 2k)k$, evaluate $\displaystyle\int_0^1 r\,dt$ (5 marks)

QUESTION TWO (20 MARKS)

a) If $\bar{a} = 2i - 3j + 4k$, $\bar{b} = i - 2j + 3k$ and $\bar{c} = 2i + j + 2k$. Find   (12 marks)

    i) $\bar{b} \times \bar{c}$

    ii) $\bar{a} . (\bar{b} \times \bar{c})$

    iii) $\bar{a} \times (\bar{b} \times \bar{c})$

    iv) $\bar{b} (\bar{a} . \bar{c}) - \bar{c} (\bar{a} . \bar{b})$

b) Find the volume of the parallelepiped whose edges are given by $\bar{a} = 2i - 3j + 4k$, $\bar{b} = i + 2j - k$ and $\bar{c} = 3i - j + 2k$ (4 marks)

c) Determine the value of m such that the vectors $\bar{a} = 2i + j + 4k$, $\bar{b} = 3i + 2j + mk$ and $\bar{c} = i - 4j + 2k$ are coplanar.   (4 marks)

QUESTION THREE (20 MARKS)

a) Consider the matrix $A = \begin{bmatrix} 2 & 2 \\ 5 & -1 \end{bmatrix}$ (8marks)

     i. Find characteristic polynomial of A.

     ii. Find all the eigenvalues of A and their corresponding eigenvectors

b) Solve the following system using Cramer's rule   (12 marks)

$$x + 3y + 2z = -13$$
$$2x - 6y + 3z = 32$$
$$3x - 4y - z = 12$$

QUESTION FOUR (20 MARKS)

a) Solve the following system of linear equation using LU decomposition.

$$x_1 + 2x_2 + 3x_3 = 5$$
$$2x_1 - 4x_2 + 6x_3 = 18$$ (14 marks)
$$3x_1 - 9x_2 - 3x_3 = 6$$

b) Find LU decomposition of the matrix

$$A = \begin{bmatrix} 1 & 4 & -3 \\ -2 & 8 & 5 \\ 3 & 4 & 7 \end{bmatrix}$$ (6 marks)

QUESTION FIVE (20 MARKS)

a) If $\underline{w} = 3ti - t^2 j$, and $\underline{z} = 2t^2 i + 3j$, verify the results   (10 marks)

     i) $\dfrac{d}{dt}(\underline{w} \bullet \underline{z}) = \underline{w} \bullet \dfrac{dz}{dt} + \dfrac{dw}{dt} \bullet \underline{z}$

     ii) $\dfrac{d}{dt}(\underline{w} \times \underline{z}) = \underline{w} \times \dfrac{dz}{dt} + \dfrac{dw}{dt} \times \underline{z}$

b) Prove that the following function is a linear transformation.   (10 marks)

$$T : R^2 \rightarrow R^2 \text{ With } T\begin{bmatrix} x \\ y \end{bmatrix} = T\begin{bmatrix} x+y \\ y \end{bmatrix}$$