UNIVERSITY OF NAIROBI

SCHOOL OF COMPUTING AND INFORMATICS

AUTOMATA THEORY CSC 222 SUBBMITED BY EXAM DAY JUNE , 2017

TIME: I HOUR

TOTAL 50 MARKS   **ANSWER ALL QUESTIONS**

Each question carries equal 5 marks

1. Suppose there are 50 marbles of four different colours in a sack, if exactly 8 marbles are red, show that there are at least 14 of the same colour. According to the Pigeon-hole principle

2. Using pumping lemma Show that $L = \{a^n : n$ is a prime number$\}$ is not regular.

3. Using Myhill-Nerode Theorem verify whether $L = \{a_nb_n : n \geq 0\}$ is regular or not

4. a) Give examples for Moore and mealy models of finite automata with outputs.
   b) Differentiate between Chomsky's Normal Form (CNF) and Greibach Normal Form (GNF).

5. Prove that the grammar

$$S \rightarrow aB \mid ab,$$
$$A \rightarrow aAB \mid a,$$
$$B \rightarrow ABb \mid b$$

is ambiguous.

6. Given a CFG as

$$G = (\{S, A, B, C, E\}, \{a, b, c\}, P, S)$$

with production P given by
$$S \rightarrow AB$$
$$A \rightarrow a$$
$$B \rightarrow b$$
$$B \rightarrow C$$
$$E \rightarrow c / \lambda$$

Obtain $L(G)$ and obtain an equivalent grammar $L(\hat{G})$ by eliminating useless terminals and productions.

7. Convert the grammar with productions

$$S \rightarrow abAB,$$
$$A \rightarrow bAB \mid \lambda,$$
$$B \rightarrow BAa \mid A \mid \lambda$$
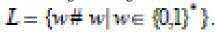
into Chomsky Normal Form.

8. Consider the set of all strings over $\{a, b\}$ with no more than twice as many a's as b's:

$$\{x \in \{a, b\}^* \mid \# a(x) \leq 2\# b(x)\}$$

(i) Give a CFG for this set, and prove that it is correct.
(ii) Give a PDA for this set. Show sample runs on the input strings

aabbaa, *aaabbb* and *aaabaa*.

9. Design a Turing machine which recognizes the language
$$L = \{w \# w \mid w \in \{0,1\}^*\}.$$

10. Design a deterministic Turing machine (DTM) to accept the language

$$L = \{a^i b^i c^i \mid i \geq 0\}.$$