

**KABARAK**



**UNIVERSITY**

**UNIVERSITY EXAMINATIONS**

**2013/2014 ACADEMIC YEAR**

**FOR THE DEGREE OF BACHELOR OF SCIENCE IN  
ECON/MATH**

**MATHS 111: VECTOR GEOMETRY**

**DAY: TUESDAY**

**DATE: 12/8/2014**

**TIME: 9:00AM – 11:00AM**

**STREAM: Y1S1**

**Instructions:**

Attempt question one and any other two question.

**Question 1 (30 marks)**

1. (a) Differentiate between a vector and a scalar giving an example of each case  
(2mks)
- (b). Let  $u = 4i + 5j + 2k$  and  $v = 2i - j + 3k$   
Find each of the following
- (i)  $2u - v$  (2mks)
- (ii)  $2|u + v|$  (3mks)
- (c). If  $c$  is a scalar and  $v$  a vector show that for any vectors in  $\mathcal{R}^3$  &  $\mathcal{R}^2$  then  
 $||cv|| = ||c|| ||v||$  (4mks)
- (d). If  $v = (2,2,1)$ , find the length of  $v$  and hence the acute vector in opposite direction  
of  $v$ : (4mks)

(e). Let  $m$  be the mid point of a line segment  $PQ$ . Let  $O$  be a point not on the line  $PQ$ . Show that:

$$\vec{OM} = \frac{1}{2} \vec{OP} + \frac{1}{2} \vec{OQ} \quad (4\text{mks})$$

(f). Let  $u$  and  $v$  be two vectors in  $\mathcal{R}^2$  &  $\mathcal{R}^3$ . Let  $\theta$  be the angle between  $u$  and  $v$  then show that:  $u \cdot v = \|u\| \|v\| \cos \theta$  and hence if  $u = (3,1,2)$  and  $v = (1,4,3)$  find the angle between  $u$  &  $v$ : (5mks)

(g). Find the angle between two planes  $x + 2y - z = 6$  and  $3x + 2y - z = 7$  (3mks)

(h). Find the equation of a plane with normal vector  $n = (1,2,3)$  containing the point  $(2,-1,5)$  (3mks)

### **Question 2 (20 marks)**

2. (a) Show that for any set of vectors  $u$  &  $v$  then  $u \times v = -v \times u$  (4mks)

(b) Find a vector orthogonal to both  $u = (1,3,2)$  and  $v = (4,0,1)$ : (3mks)

(c) A triangle  $ABC$  has vertices on  $A(1,2,2)$   $B(3,4,5)$  and  $C(5,6,4)$ . Find its area. (5mks)

(d) A parallelepiped has vectors on its edges as following  $u = (2,3,1)$   $v = (3,4,3)$  and  $w = (4,5,6)$ . Find its volume: (5mks)

(e) For  $U = 2i + j - k$ ,  $V = 3i + 2j + k$ . Find  $U \times V$  (3mks)

### **Question 3 (20 marks)**

3. (a). Given two vectors In  $\mathfrak{R}^n$   $\mathbf{u}$  and  $\mathbf{v}$ . show that

$$\mathbf{u} \cdot \mathbf{v} = \frac{1}{4} (|\mathbf{u} + \mathbf{v}|^2 - |\mathbf{u} - \mathbf{v}|^2) \quad (4\text{mks})$$

(b) Let  $\mathbf{u} = (1,2,-5)$   $\mathbf{v} = (3,-1,2)$  and  $\mathbf{w} = (2,0,3)$ . Find each of the following.

(i)  $(2\mathbf{u} + \mathbf{v}) \cdot \mathbf{w}$  (3mks)

(ii)  $(\mathbf{u} - 3\mathbf{v}) \cdot \mathbf{w}$  (3mks)

(c) Find the cosine of angle between  $\mathbf{u}(1,2,3)$  and  $\mathbf{v}(3,-2,1)$  (4mks)

(d) Determine the value of  $a$  so that  $\mathbf{A} = 2\mathbf{i} + a\mathbf{j} + \mathbf{k}$  and  $\mathbf{B} = 4\mathbf{i} - 2\mathbf{j} - 2\mathbf{k}$  are perpendicular: (3mks)

(e) Find the projection of  $\mathbf{A} = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$  on vector  $\mathbf{B} = 4\mathbf{i} - 4\mathbf{j} + 7\mathbf{k}$  (3mks)

#### **Question 4 (20 marks)**

4. (a). Given a set of vector  $\mathbf{u} = 2\mathbf{i} + \mathbf{j} + \mathbf{k}$  and  $\mathbf{v} = 3\mathbf{i} - 2\mathbf{j} - \mathbf{k}$

Find (i)  $\mathbf{u} \times \mathbf{v}$  (2mks)

(ii) The sine of the angle between  $\mathbf{u}$  and  $\mathbf{v}$  (5mks)

(b) Show that  $\mathbf{a} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ ,  $\mathbf{b} = 4\mathbf{i} + 5\mathbf{j} + 6\mathbf{k}$  and  $\mathbf{c} = 7\mathbf{i} + 8\mathbf{j} + 9\mathbf{k}$  all lie on the same plane: (5mks)

(c) State and prove the distributive Law of cross products: (4mks)

(d) If  $2x + 4y - 5z$  is an equation of a plane. Find the normal vector and a point on this plane: (4mks)

#### **Question 5 (20 marks)**

5. (a). Find a unit vector parallel to a resultant vector  $r_1 = 2i + 4j - 5k$ ,  
 $r_2 = i + 2j + 3k$  (4mks)
- (b). Show if  $i + 2j + 3k = u$  and  $3i + j + k = v$  are orthogonal. (4mks)
- (c) Write a parametric equation for a plane whose Cartesian equation is  
 $X + 2y - z = 7$  (4mks)
- (d).  $2x + 4y - 5z = 11$ . Is an equation of a plane. Find the normal vector and a point  
on this plane (4mks)
- e). Find the volume of a parallelepiped determined by the vectors  $(-1, 2, 3)$ ,  $(2, -1, 1)$ ,  
 $(3, -2, 3)$  (4mks)