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**JOMO KENYATTA UNIVERSITY**

**OF**

**AGRICULTURE AND TECHNOLOGY**

**UNIVERSITY EXAMINATIONS 2014/2015**

**FOURTH YEAR FIRST SEMESTER EXAMINATION FOR THE DEGREE**

**OF BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE**

**STA 2498: DECISION THEORY AND BAYESIAN INFERENCE II**

**DATE: AUGUST 2015 TIME: 2 HOURS**

**INSTRUCTIONS: ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS**

**QUESTION ONE (COMPULSORY) (30 MARKS)**

1. i) Define Bayesian Inference. [2 marks]

ii) Three manufacturers supply clothing to a retailer.

60% of the stock comes from manufacture 1, 30% from

manufacturer 2 and 10% from manufacturer 3. 10% of the

clothing from manufacturer 1 is faulty, 5% from manufacturer

2 is faulty and 15% from manufacturer 3 is faulty. What is the probability that a faulty garment comes from manufacturer 3?

 [4 marks]

 iii) The overall risk of breast cancer is any 45 year old woman

 regardless of any mammogram result is 0.1% (one in a

 thousand). Assume that the mammography is 80% sensitive

 and 95% specific. What is the probability that this woman

 actually has breast cancer? [4 marks]

1. Differentiate between:
2. Informative and non-informative prior [2 marks]
3. Conjugate and non-conjugate prior [2 marks]
4. Frequentist inference and Bayesian Inference [4 marks]
5. Suppose that a sample comes from the poison distribution

with dpf .

  for x = 0, 1,2 - - -

1. Determine the likelihood function [2 marks]
2. Computer the posterior distribution [3 marks]
3. Hence determine the Bayes estimator [2 marks]
4. Suppose that Y is distributed as poison random variable.

Derive the Jefferys Prior f( ) that is invariant under

reparameterization. [5 marks]

**QUESTION TWO (20 MARKS)**

1. An insurance company is monitoring the length of time staff

take to pick a call after the first ring. It is assumed that the time

follows on exponential distribution with parameter $θ$. 10 calls are monitored at random and the average mean response is calculated

as 3.672 seconds.

1. Show that the Gamma distribution is the conjugate

prior distribution for $θ$ [4 marks]

1. Assuming that the prior distribution of $θ$ has a mean of

0.315 and standard deviation 0.251, derive the posterior

distribution of $θ$ and calculate the Bayesian estimate of

$θ $under the quadratic loss. [3 marks]

1. A further 70 calls are monitored and have the same

average response time of 3.672. Calculate the Bayesian

estimate of $θ $under the quadratic loss using all data

collected. [3 marks]

1. The developer of a new complex claims that 3 out 5 buyers

will prefer a two bedroom unit, while his banker claims that it

would be more correct to say that 7 out of 10 buyers will prefer

a two bedroom unit. In previous prediction of this type, the bank

has been twice as reliable as the developer. 1f 12 of the next 15

units sold in this complex are two bedroom units, find:

1. The posterior probabilities associated with the

claims of the developer and banker. [5 marks]

1. A point estimate of the proportion of buyers who

prefer a two bedroom unit. [5 marks]

**QUESTION THREE (20 MARKS)**

1. A random sample of n students is drawn from a large population

and their weights are measured. The average weight of the n - sample student is  = 150. Assume the weights in the population are normally distributed with unknown mean $θ$ and known standard deviation 20. Suppose your prior distribution for $θ$ is normal with mean 180 and standard deviation 40.

1. Derive the posterior distribution for $θ$ (your answer will

be a function of n). [6 marks]

1. For n = 10, give a 95% posterior interval for $θ$ and

a 95% posterior predictive interval for . [4 marks]

1. i) The length of life of a light bulb manufactured by a certain

 process has an exponential distribution with unknown rate $θ$.

 Suppose the prior distribution for $θ$ is a gamma distribution

with coefficient of variation 0.5. A random sample of light

bulbs is to be tested and a life time of each obtained. If the

coefficient of variation of the distribution of $θ$ is reduced to

0.1, how many light bulbs need to be tested.

(Coefficient of variation = , Var (x) = K$ θ$2,

E (x) = K$ θ$ for Gamma distribution) [5 marks]

ii) If the coefficient of variation refers to ɸ (precision) instead

 of $θ$ how would the solution in b(i) change. [5 marks]

**QUESTION FOUR (20 MARKS)**

1. Let X1, X2 …., Xn be N (,) random variables with the

prior for  having N (, ) .

Suppose its known from past experience that the weight loss

of a particular combination of diet and exercise programme

is normally distributed with mean of 10 kg and standard deviation

of 2 kg. A random sample of 5 persons went through this programme

and produced the following weight loss in kg.

 14, 8, 11, 7, 11

What is the point estimate of the mean? Assume  = 4 [7marks]

1. Show that the family of exponential function

f(yi/ $θ$) = a (yi) b ($θ$) exp {C ($θ$)T d (yi)}

Has natural conjugate prior distributions. [7marks]

1. Discuss how you can computer the highest posterior

density of $θ$ [6 marks]