DEDAN KIMATHI UNIVERSITY OF TECHNOLOGY

UNIVERSITY EXAMINATIONS 2014/2015

FIRST YEAR SUPPLEMENTARY/ SPECIAL EXAMINATION FOR THE DEGREE IN BACHELOR OF SCIENCE IN MECHANICAL ENGINEERING, MECHATRONIC ENGINEERING, CIVIL ENGINEERING, ELECTRICAL & ELECTRONIC ENGINEERING AND INDUSTRIAL CHEMISTRY

SMA 2172 CALCULUS I

DATE: 29[TH] SEPTEMBER 2015 **TIME : 8.30AM – 10.30AM**

INSTRUCTIONS :Answer question ONE and any other TWO questions

Question 1. (30 marks)

(a) (i) State the £ − ¢ definition of the limit of a function $f(x)$ as x tends to the point $x = x_0$ .Use this to prove that

$$\lim_{x \to 2} 2x - 1 = 3$$

(4 marks)

(ii) Given

$$f(x) = \begin{cases} 4 & if x \prec 2 \\ 1 & if x = 2 \\ (x-4)^2 & if x \succ 2 \end{cases}$$

Evaluate

$\lim_{x \to 2^-} f(x)$ and $\lim_{x \to 2^+} f(x$.

Why is the function not continuous at $x = 2$? (3 marks)

(b) State a suitable definition for the derivative of a function $f(x)$ and use it to find the derivative of $\sin 2x$. (5 marks)

(c) Calculate the gradient to the curve $x^2 - 2y^2 = 4$ at the point $(6,4)$ (2 marks)

(d) Evaluate $\int \cos^2 x dx$. (3 marks)

(e) Find the area between the curve $y = 3x - x^2$ and the $x - axis$ between the ordinates $x = 0$ and $x = 5$. (4 marks)

(f) By differentiating implicitly,find $\frac{dy}{dx}$ given that $x^3y + 3xy - 5 = 1$ (4 marks)

(g) Differentiate the function $y = \frac{1}{2x+1}$,with respect to x, from first principles. (5 marks)

Question two.(20 marks)

(a) Briefly explain the following

(i) The limit of a function $f(x)$ (1 mark)

(ii) A continuous function (2 marks)

Hence determine whether the function $f(x) = \frac{x^3 - 1}{x^2 - 1}$ is continuous at $x = 1$. (3 marks)

(b) Find the derivative of the following function from first principles

$2\sqrt{x} + \frac{1}{2\sqrt{2}}$. (4 marks)

(c) Evaluate the following

(i) $\lim\limits_{x \to 3^-} \frac{9}{(x-3)^2}$ (2 marks)

(ii) $\lim\limits_{t \to \infty} \sqrt[3]{t} + 12t - 2t^2$ (3 marks)

(d)Differentiate the following functions

(i) $f(x) = \ln(3x^2 + x)$ (2 marks)

(ii) $f(x) = 7^x$ (3 marks)

Question three:(20 marks

(a)Given that $f(x) = u(x)v(x)$ where u and v are functions of x ,prove that

$\frac{df}{dx} = v(x)\frac{du}{dx} + u(x)\frac{dv}{dx}$ (7 marks)

(b)Find the equation of the tangent to the curve $x^3 + xy^3 = 9$,and the tangent starts at the point $(1,2)$. (5 marks)

(c)Obtain a reasonable approximate value of $(2.98)^3$ (4 marks)

(d)Given $f(u) = u^3 - 3u^2 + 5u - 4 = y, and\ u(x) = x^2 + x$ .What is $\frac{dy}{dx}$? (4 marks)

Question four:(20 marks)

(a)Calculate $y', y'' y''' y^{iv} y^{v}$ where $y = 8x^3 - 6x + 5$ (4 marks)

(b)A particle moves along a long straight line in such a manner that its distance $s, (m)$

From a fixed point on the line at the end of $t - \sec onds$ is given by

$s = t^3 - 3t^2 - 24t$

(i)Find its velocity when its acceleration is zero. (2 marks)

(ii)Find its acceleration when its velocity is zero. (2 marks)

(c)Sketch the following $y = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 6x + 8$. (8 marks)

(d)Find $y''(x)$; where $x^{\frac{1}{2}} + y^{\frac{1}{2}} = 1$. (4 marks)

Question five:(20 marks)

(a)Find the length of the curve given by the following parametric representation,

$x = \frac{1}{3}t^3$ and $y = t^2$. (4 marks)

(b)Find the curved surface of the solid generated by revolving the part of

$y = x^2$ from $(0,0)$ to $(\sqrt{6}, 6)$ about the $y - axis$ (4 marks)

(c)Find $\int (x^3 - 4x^2 + 10)dx$ (3 marks)

(d)Prove that if

$u = x^3 \tan^{-1} \frac{y}{x}$ then

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 3u$$

and deduce that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 6u .$$ (9 marks)