

**W1-2-60-1-6**

**JOMO KENYATTA UNIVERSITY**

**OF**

**AGRICULTURE AND TECHNOLOGY**

**UNIVERSITY EXAMINATIONS 2015/2016**

**THIRD YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN PHYSICS/ CONTROL AND INSTRUMENTATION**

 **SPH 2305: MATHEMATICAL PHYSICS I**

**DATE: DECEMBER 2015 TIME: 2 HOURS**

**INSTRUCTIONS: ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS**

**The following Information may be Useful:**

1. Generating functions

 for legendre polynomials

  for Hermite polynomials

1. Transformation equations

, , Ƶ = Ƶ for the cylindrical co-ordinate system

, , Ƶ =  for the spherical co-ordinate system.

**QUESTION ONE (30 MARKS)**

1. The Vectors A, B, and C are given as

A = i + j, B = 2i -3j +k and C = 4j -3k

Determine

1. A x (B x C) **(3 marks)**
2. The volume of the parallepiped of sides A, B, and C **(3 marks)**
3. The displacement of particle at any time t is given by the vector

r = (t3 + 2t)i -3e-2t j + 2sin 5t k

Find the particle’s

1. Initial velocity **(2 marks)**
2. Initial speed **(2 marks)**
3. Initial acceleration **(2 marks)**
4. Using vector algebra, prove that the line joining the mid-points

of two sides of a triangle is parallel to the third side and has half

its length. **(4 marks)**

1. Evaluate the integral  along
2. Curve y = x+1 **(3 marks)**
3. The parabola x = t, y = t2 +1 **(3 marks)**
4. Prove that the divergence of a curl of any vector A is zero **(4 marks)**
5. Use the generating function for Legendre polynomials, Pn(x),

to show that:



 **(4 marks)**

**QUESTION TWO (20 MARKS)**

1. For a transformation from a Cartesian to a spherical co-ordinate

system, determine the:

1. Scale factors of the transformation  **(3 marks)**
2. Arch length ds2 **(2 marks)**
3. Jacobian of the transformation **(2 marks)**
4. Volume element dV **(2 marks)**
5. Volume of a sphere of radius r **(2 marks)**
6. i) State Green’s Theorem in the plane and **(2 marks)**

ii) Verify it for the closed integral **(1 mark)**

 

 taken over the region enclosed by y=x2 and y2 = x **(5 marks)**

1. Consider the complex function F(Ƶ) = 

Determine its singularities and their order. **(3 marks)**

**QUESTION THREE (20 MARKS)**

1. i) Use the generating function for a Hermite polynomial Hn(x)

 to show that:

  = 2n Hn-1(x) **(3 marks)**

ii) Hence determine H1 (x) and H2 (x) **(4 marks)**

1. Consider a transformation from the Cartesian to the cylindrical

co-ordinate system. Determine:

1. Scale factors of the transformation **(3 marks)**
2. Jacobian of the transformation **(2 marks)**
3. Volume element **(1 mark)**
4. Volume of a cylinder of height h **(2 marks)**
5. A force field is given by

F = (3x – 4y +2 Ƶ)i + (4x +2y – 3Ƶ2)j + (2x Ƶ - 4y2 + Ƶ3)k

Determine the work down by this force in moving a particle

once around an eclipse described by the parametric equations

,  where 

. **(5 marks)**

**QUESTION FOUR (20 MARKS)**

1. A function of a complex number is given as

f(Ƶ) = Ƶ3+ Ƶ +1

1. Decompose it into the real and complex parts u(x,y)

and v(x,y) respectively. **(2 marks)**

1. Show that u(x,y) and v(x,y) are harmonic functions. **(2 marks)**
2. Given the complex number 3 + 3i

i) Express it in the polar form **(2 marks)**

ii) Use De Moivre’s theorem to evaluate (3 + 3i)10 **(3 marks)**

1. Consider Bessel’s differential equation:

x2y” + xy’ + (x2 – n2)y = 0

i) Write its general solution. **(1 mark)**

ii) Use the power series method to determine its series solution. **(10 marks)**