SOUTH EASTERN KENYA UNIVERSITY

UNIVERSITY EXAMINATIONS 2016/2017

FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION (ARTS) ,BACHELOR OF EDUCATION (SCIENCE), BACHELOR OF SCIENCE (ECONOMICS AND STATISTICS) BACHELOR OF ECONOMICS.

SMA 203: LINEAR ALGEBRA 1

DATE: 7TH DECEMBER, 2016 **TIME: 1.30-3.30PM**

INSTRUCTION TO CANDIDATES

Answer **Question One** and **Any Two** other Questions

Question 1 (30 Marks)

(a) Define the following terms:

(i) basis of a vector space V **(2marks)**

(ii). a vector subspace **(2marks)**

(iii). kernel of vector. **(2marks)**

(b) Let T be the linear transformation from R_2 to R_2 such that $(1,2)T = (1,0)$ $(3,-1)T = (2,2)$; compute $(2,4)T$ **(4marks)**

(c). The matrix (with respect to the natural basis) of a rotation of the plane through θ radians is A= $\begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix}$

(i). Write down A^T **(2marks)**

(ii) Compute AA^T **(3marks)**

(d). Determine the dimension of the vector space spanned by the vectors $\vec{u} = (2,-1,3)$, $\vec{v} = (4,0,6)$ and $\vec{w} = (8,-2,-3)$. **(5 marks)**

(e). Show whether the vector $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$ is in the column space of the matrix $\begin{pmatrix} 2 & 1 \\ 2 & 5 \end{pmatrix}$. **(3marks)**

(f). Let $f:\Re^4 \to \Re^3$ be multiplication by $\begin{pmatrix} 4 & 1 & -2 & -3 \\ 2 & 1 & 1 & -4 \\ 6 & 0 & -9 & 9 \end{pmatrix}$. Which of the following vectors are in the null space of f, $\vec{u} = (3,-8,\ 2,0)$, $\vec{v} = (0,0,0,1)$ and $\vec{w} = (0,-4,1,0)$?

(4 marks)

(g). Let T be a mapping from a vector space V into a vector apace W. Explain what is meant by: **(i)** Range of T **(1 mark)**

(ii) T is a linear transformation **(2 mark)**

Question 2 (20 Marks)

(a) .Determine whether the vectors (1,1,1), (2,2,0) and (3,0,0) span $\Re^3$. **(4 marks)**

(b) (i). Let V be a vector subspace of $\Re$ of polynomials of degree less than or equal to 2. Check whether or not the following vectors in V are linearly independent;

$x^2 + 3x + 2,\ 2x^2 + x,\ 2x^2 - x - 1.$ **(5 marks)**

(ii) Show that $\vec{w} = (9,2,7)$ is a linear combination of $\vec{u} = (1,2,-1,)$ and $\vec{v} = (6,4,2)$.

(4 marks)

(c) Find the kernel, range, rank and nullity for the linear function

$f:\Re^2 \to \Re^3$ given by f(x, y) = (x, 2y, x+y). **(7 marks)**

Question 3 (20 Marks)

(a) Let T be the linear transformation, clockwise direction of the plane through 90^0 and let S be the linear transformation, a reflection in the y axis. Compute $(x,y)\ (T^3 - 3T^2S^2 + 3S^2T)$

(8marks)

(b) Find an ordered basis β for R_2 such that $(1,0)C_\beta = (1,2)$ and $(0,1)C_\beta = (3,4)$ **(6marks)**

(c) Given that β is the ordered basis $\{(1,2),\ (0,3)\}$ for R_2 .

Compute $(x_1, x_2)C_\beta$. **(2marks).**

(d) Compute $[7\begin{pmatrix}1 & 2\\ 3 & 4\\ 5 & 6\end{pmatrix}]\begin{pmatrix}1 & 0 & 6 & 1\\ 6 & 1 & 4 & 0\end{pmatrix}$ **(4marks)**

Question 4 (20 Marks)

(a) Compute AA^T if $A=\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}$ **(3marks)**

(b) If β is the ordered basis for R_2 find x if $x\ C_\beta = (5,2)$; that is $(5,2)\,C_\beta^{-1}$ **(3marks)**

c). For the matrix $G=\begin{pmatrix}3 & 9 & 6\\ 0 & 0 & 0\\ 2 & 6 & 4\end{pmatrix}$, find the rank ,

(2marks)

I. Classify as singular or nonsingular, **(2marks)**

II. Describe the row and column spaces geometrically **(2marks)**

(d) Given the linear system below, use Gaussian elimination method to solve for w, x, y, z .

$$w - x + 2y - z = -8$$
$$2w - 2x + 3y - 3z = -20$$
$$w + x + y + 0z = -2$$
$$w - x + 4y + 3z = 4$$

(8marks)

Question 5 (20 Marks)

(a). Find a basis and the dimension of the vector space of matrices $\begin{pmatrix}a & b\\ c & d\end{pmatrix}$ subject to each of the following conditions: **(i).** $a, b, c, d \in \Re$ **(4 marks)**

(ii). $a - b + 2c = 0 \quad and \quad d \in \Re$ **(3 marks)**

(iii). $a + b + c = 0, \ a + b - c = 0 \quad and \quad d \in \Re$ **(2 marks)**

(b). Determine whether T is a linear transformation in the following:

$T : (x, y, z) \rightarrow (x + z, y - x, z + y - x), \ that \ is \ T : \Re^3 \rightarrow \Re^3$ **(6 marks)**

(c). Use Cramer's rule to solve the following system of linear equations.

$$2x + 3y - z = 1$$
$$3x + 5y + 2z = 8$$
$$x - 2y - 3z = 1$$

(5 marks)